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Determination and Interpretation of Activation Energy Using Accelerated-Test Data

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SUMMARY & CONCLUSIONS

The activation energy is an indication of effect that the covariate (temperature, humidity, etc.) has on the life of the product. An incorrect assumption on the value of the activation energy can lead to an incorrect deduction regarding the effect that the covariate has on the product, as well as an erroneous calculation of the acceleration factor. Estimating the activation energy based on the accelerated-test data removes all of the guesswork when it comes time to decide on a value. The idea of estimating the activation energy based on data is not exactly a new idea, but now this process can be conducted more easily. In the past, the process was long and tedious and involved a lot of iterative calculations. But now, the ability to estimate the activation energy based on collected accelerated-test data has been implemented within a software package, ALTA™, which is expressly designed for accelerated life testing analysis.

The activation energy can be determined by simply making it one of the unknown parameters that need to be estimated. In doing so, the problems which arise from selecting an activation energy have been removed, and the data alone dictates the value. To illustrate this, accelerated life test data was collected on thermally stressed hard drive systems and analyzed using the Arrhenius-Weibull model. The parameters of the Arrhenius model are then estimated using maximum likelihood theory. Once the parameters of the life-stress relationship are obtained, the activation energy can then be easily estimated, along with the acceleration factor. The activation energy now does not just represent a particular failure mode, but the entire collection of failure modes for which the test was conducted.

1. INTRODUCTION

There are many factors to consider when conducting an analysis of accelerated-test data. One factor that is often overlooked or misrepresented is the activation energy. When considering accelerated life tests, the activation energy represents the magnitude of effect that the applied stress will have on the product under test. A large activation energy indicates that the applied stress will have a large effect on the life of the product. Understanding activation energy is one

thing, but obtaining an accurate value is another. Currently, the published values of activation energy relate to a particular material, technology, or failure mode, but how this value relates to an entire system, which may be composed of many different materials, is unknown. The published values can also be misleading. These values are constant for a particular stress or failure mode, but in reality the activation energy may actually vary from one production lot to another and from one version of the product to another. In addition, the activation energy can vary from one failure mode to another, even for the same product [1]. Therefore, it may not be practical to use the published values. It is also possible to use an activation energy value based on previous information, but even this does not take into consideration any design changes that might affect its value.

One possible method to alleviate the problem of selecting the most representative activation energy is to estimate the value based on collected data. Utilizing maximum likelihood theory, the parameters of the Arrhenius model are solved while using the Weibull distribution as the underlying life distribution. Once the parameters have been estimated, the activation energy and acceleration factor can be easily calculated.

2. NOMENCLATURE

<i>b</i>	Weibull shape parameter
<i>h</i>	Weibull scale parameter
<i>A</i>	Unknown thermal constant
<i>B, C</i>	Parameters of the Arrhenius model
<i>E_a</i>	Activation energy (eV)
<i>E_{ai}</i>	Activation energy of subpopulation <i>i</i>
<i>AF</i>	Acceleration factor
<i>K</i>	Boltzman's constant ($8.617385 \cdot 10^{-5}$ eV/K)
<i>t</i>	Time
<i>NF</i>	Number of failures
<i>NS</i>	Number of suspensions
<i>T</i>	Temperature in absolute scale (Kelvin)
<i>T_{use}</i>	Use temperature level (Kelvin)
<i>T_{accel}</i>	Accelerated temperature level (Kelvin)
<i>f(t)</i>	Probability density function
<i>T_F</i>	Exact time-to-failure
<i>T_S</i>	Right censored time-to-failure

<i>pdf</i>	Probability density function
p_i	Proportion of subpopulation i
$F_i(t)$	Cumulative distribution function of subpopulation i
m	Total number of subpopulations
<i>MTTF</i>	Mean-time-to-failure of whole population
<i>MTTF_i</i>	Mean-time-to-failure of subpopulation i
r	Speed of reaction

3. ASSUMPTIONS

The following assumptions are made prior to the analysis:

1. The underlying life distribution has a common shape parameter across different stress levels. This assumption is valid when similar failure modes are accelerated.
2. The initial life parameters are constant over at least most of the life of the product.

4. LIFE-STRESS RELATIONSHIP

Accelerated life models are composed of a life-stress relationship and an underlying life distribution, usually Weibull, lognormal, or exponential. Once the accelerated life model has been defined, the objective is then to estimate the parameters of the failure distribution and life-stress relationship [3]. There are many ways to estimate the parameters, including graphical methods and rank regression, but the focus in this paper will be maximum likelihood estimation (MLE). The likelihood function can be written as

$$L = \prod_{i=1}^{NF} f(T_{F,i}) \prod_{j=1}^{NS} R(T_{S,j}) \quad (1)$$

Eq. (1) can be simplified by taking the natural logarithm of both sides. The log-likelihood function is given by

$$\ln(L) = \Lambda = \sum_{i=1}^{NF} [\ln(f(T_{F,i}))] + \sum_{j=1}^{NS} [\ln(R(T_{S,j}))] \quad (2)$$

The next section describes how Eq. (2) becomes a function of the applied stress by expressing the *pdf* and reliability functions in terms of stress.

4.1 Arrhenius Model

When temperature is used as the single stress covariate, the Arrhenius model can be an appropriate model to use. The Arrhenius reaction rate equation is given by the following

$$r = Ae^{-\frac{E_a}{K*T}}$$

Now assuming that the life of a product is proportional to the inverse of the rate of reaction [2], the Arrhenius life-stress relationship is given by

$$L(T) = Ce^{\frac{E_a}{K*T}} \quad (3)$$

Furthermore, the *pdf* of the 2-parameter Weibull distribution is given by

$$f(t) = \frac{b}{h} \left(\frac{t}{h}\right)^{b-1} e^{-\left(\frac{t}{h}\right)^b}$$

The scale parameter (characteristic life) is h . The Arrhenius-Weibull model *pdf* is then obtained by setting $h = L(T)$ [2],

$$f(t, T) = \frac{b}{Ce^{\frac{E_a}{K*T}}} \left(\frac{t}{Ce^{\frac{E_a}{K*T}}}\right)^{b-1} e^{-\left(\frac{t}{Ce^{\frac{E_a}{K*T}}}\right)^b} \quad (4)$$

The reason for doing this is that a life characteristic or characteristic point needs to be selected (or created) that allows for the relationship to be generated across the different stress levels. This life characteristic must be a function of stress, and for the Weibull distribution the scale parameter, h , has been chosen. Some typical life characteristics are presented in Table 1 [2].

Distribution	Parameters	Life Characteristic
Weibull	b, h	Scale Parameter (h)
Exponential	l	Mean Life (1/l)
Lognormal	m, s	Median (\tilde{T})

**Table 1: Typical life characteristics
(b and s are usually assumed to be constant)**

But in Eq. (4), the activation energy, E_a , must be known a priori and this is rarely the case. So another parameter, B , is introduced, where

$$B = \frac{E_a}{K}$$

The Arrhenius life-stress relationship, Eq. (3), can also be rewritten as

$$L(T) = Ce^{\frac{B}{T}} \quad (5)$$

So now the Arrhenius-Weibull model *pdf* can be written as

$$f(t, T) = \frac{b}{Ce^{\frac{B}{T}}} \left(\frac{t}{Ce^{\frac{B}{T}}}\right)^{b-1} e^{-\left(\frac{t}{Ce^{\frac{B}{T}}}\right)^b}$$

The Arrhenius reliability equation is given by

$$R(t, T) = e^{-\left(\frac{t}{Ce^{\frac{B}{T}}}\right)^b}$$

Then the Arrhenius-Weibull likelihood function is given by the following equation.

$$\Lambda = \sum_{i=1}^{NF} N_i \ln \left[\frac{b}{Ce^{\frac{B}{T}}} \left(\frac{t_i}{Ce^{\frac{B}{T}}} \right)^{b-1} e^{-\left(\frac{t_i}{Ce^{\frac{B}{T}}} \right)^b} \right] - \sum_{i=1}^{NS} N'_i \left(\frac{t_i}{Ce^{\frac{B}{T}}} \right)^b \quad (6)$$

The parameters which need to be estimated are **b**, **B**, and **C**. Using Eq. (6), the parameters can be estimated using MLE. Once all of the parameters have been estimated, determining the activation energy is elementary. The activation energy is then simply,

$$E_a = B * K$$

Extensive coverage of the Arrhenius model and maximum likelihood estimation is provided by Nelson [1].

4.1.1 Significance of B

The significance of the parameter B can be seen if the life-stress relationship is linearized by taking the natural logarithm of both sides in Eq. (5)

$$\ln(L(T)) = \ln(C) + \frac{B}{T} \quad (7)$$

In Eq. (7), ln(C) is equal to the intercept, the inverse of the stress (temperature) is the variable, and B is the slope of the line. But B is not the slope of the life versus stress; it is the slope of the life versus the inverse of the stress [2]. An example of a life vs. stress plot, also referred to as an Arrhenius plot, is shown in Figure 1.

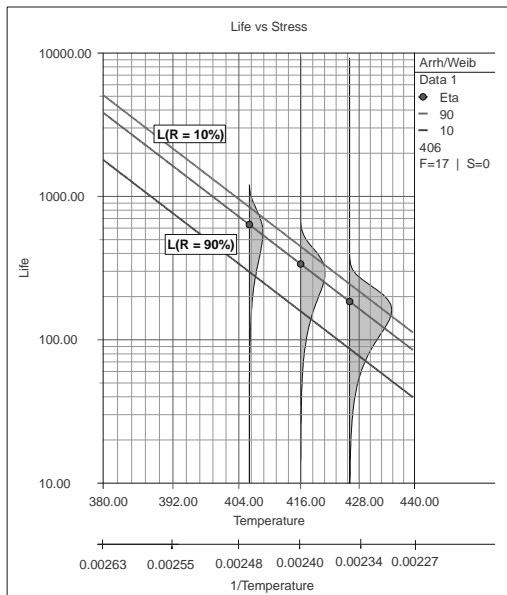


Figure 1: Example of life vs. stress plot with reciprocal and non-reciprocal scales

It would seem from Figure 1 that B, and therefore the activation energy, is negative. This is due to that fact that the x-axis is actually a reciprocal scale. The x-axis is plotted on a reciprocal scale simply for practical purposes. It is just easier to read the life at 404K than at 0.00248 (the reciprocal of 404). Figure 1 actually illustrates that the B is positive. As the reciprocal of the stress increases, the stress decreases. If the activation energy is small then you can expect the life of the product to change slowly as the temperature changes. This can also be seen in B as shown in Figure 2.

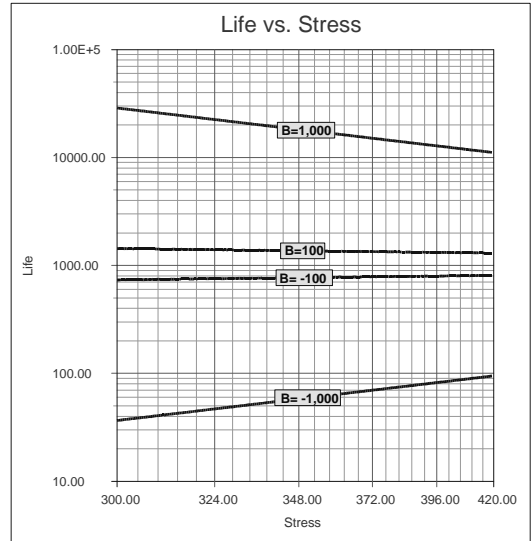


Figure 2: Behavior of the parameter B

In Figure 2 you can also see that the magnitude of B determines the slope of each line. In addition, B can also take on negative values. A negative B would indicate that the life of the product increases as the stress increases. But is this plausible? Instinct tells us that as stress is increased life should decrease. In the end, even the most obvious assumption could be wrong and should be verified. After all, in plasma filled bulbs low temperatures are the high stressing condition as compared room (use) temperatures [2]. This situation assumes the stress versus life relationship is fairly linear over the temperature range.

4.1.2 Acceleration Factor

Once the activation energy has been estimated, the acceleration factor can be determined next. The acceleration factor is most often referred to as the ratio of the life between use stress level and some higher (accelerated) test stress level, or

$$AF = \frac{L_{use}}{L_{accel}}$$

If the activation energy is known apriori (rarely the case), then the acceleration factor can be written as

$$AF = e^{\left(\frac{E_a}{K * T_{use}} - \frac{E_a}{K * T_{accel}} \right)} = e^{\frac{E_a}{K} \left(\frac{1}{T_{use}} - \frac{1}{T_{accel}} \right)} \quad (8)$$

However, Eq. (8) can be rewritten as

$$AF = \frac{C e^{\frac{B}{T_{use}}}}{C e^{\frac{B}{T_{accel}}}} = \frac{e^{\frac{B}{T_{use}}}}{e^{\frac{B}{T_{accel}}}} = e^{\left(\frac{B}{T_{use}} - \frac{B}{T_{accel}}\right)} \quad (9)$$

From Eq. (9) it is possible to estimate the value of the acceleration factor based on the estimated parameter B (which includes the activation energy), as opposed to assuming a value for the activation energy. It can also be seen from Eq. (8) and (9) that the AF depends only on the E_a or B . Therefore, an incorrect assumption on either of these values will have a large effect on the value for acceleration factor. In reference to acceleration factor, Nelson [1] states, "...often its value is a company tradition with unknown origins." This is probably due to the fact that a value for the activation energy was assumed. But no longer do the activation energy and acceleration factor need to have "unknown origins."

5. MULTIPLE FAILURE MECHANISMS

The previous sections discussed data with primarily a single failure mode. But this is not always going to be the case. In general, multiple failure mechanisms are going to be possible for a given product. Now it is possible to associate a single activation energy and acceleration factor for all failure modes of a component or system, but as Nelson [1] points out, "this is usually crude." So there must be a better way.

When multiple failure mechanisms are present, the overall activation energy corresponds to the minimum energy required to activate the weakest failure mechanism. Even though this activation energy is an inherent characteristic of an item, it is random in reality due to the variation from one item to another in materials processing, manufacturing, assembling, packaging, shipping, installation etc. The mixed life distribution can be used to describe the combined effect of multiple failure mechanisms. For example, the cumulative distribution function (CDF) for a mixed life distribution with m subpopulations under normal temperature conditions is given by

$$F(t) = \sum_{i=1}^m p_i F_i(t) \quad (10)$$

where,

$$\sum_{i=1}^m p_i = 1$$

The corresponding mean life is written as

$$MTTF = \sum_{i=1}^m p_i MTTF_i \quad (11)$$

The equation for the AF without the activation energy is as follows

$$e = e^{\frac{1}{k} \left(\frac{1}{T_{use}} - \frac{1}{T_{accel}} \right)}$$

Then the Arrhenius acceleration factor can be written as

$$AF = e^{E_A}$$

The relationship between the mean time to failure under accelerated temperature conditions, T_{accel} , and that under normal temperature conditions, T_{use} , can be expressed as

$$MTTF_{accel} = MTTF_{use} / AF = MTTF_{use} e^{-E_A} \quad (12)$$

Substituting Eq. (12) into Eq. (11) and simplifying yields

$$MTTF_{accel} = \sum_{i=1}^m (p_i e^{E_{Ai} - E_A}) MTTF_{i,accel} \quad (13)$$

This indicates that the subpopulation proportion, after being exposed to high temperature, changes to

$$p'_i = (p_i e^{E_{Ai} - E_A})$$

Note that the following relation should remain true,

$$\sum_{i=1}^m p'_i = \sum_{i=1}^m (p_i e^{E_{Ai} - E_A}) = 1$$

or

$$E_A = \frac{\text{Log} \left(\sum_{i=1}^m p_i e^{E_{Ai}} \right)}{\text{Log}(e)} \quad (14)$$

Eq. (14) gives the relationship between the activation energy of the overall population and that of the subpopulation.

6. EXAMPLE

To illustrate the process of estimating the activation energy, 399 hard drive systems were thermally stressed at 308K and 328K for 1032 hours and three failures were observed. The collected data is shown in Table 2, where "F" indicates that the observation has failed, while "S" indicates a right censored (suspended) observation.

Number in Group	F or S	Time (hours)	Temperature (K)
1	F	48	308
1	S	60	308
197	S	1032	308
1	S	13	328
1	F	16	328
1	S	24	328
1	S	38	328
1	S	65	328
1	F	73	328
1	S	79	328
1	S	98	328
1	S	250	328
1	S	361	328
190	S	1032	328

Table 2: Data for thermally stressed hard drive systems

Arrhenius is selected as the life-stress relationship and Weibull as the underlying life distribution. It is also assumed that the use stress level is 308K. The estimated parameters obtained using ALTA™ are:

$$b = 0.3074; B = 11674.2; C = 1.0506E-6$$

Therefore, the estimated value of the activation energy is equal to

$$E_a = B * K = 11674.2 * 8.617385 * 10^{-5} = 1.0060$$

This certainly is a reasonable value and indicates that there is definitely some acceleration due to the temperature stress. Given $E_a = 1.0060$, $T_{use} = 308K$ and $T_{accel} = 328K$, the acceleration factor is estimated as follows

$$AF = e^{\frac{E_a}{K} \left(\frac{1}{T_{use}} - \frac{1}{T_{accel}} \right)} = 10.0863$$

The acceleration factor can also be seen in the Arrhenius plot shown in Figure 3.

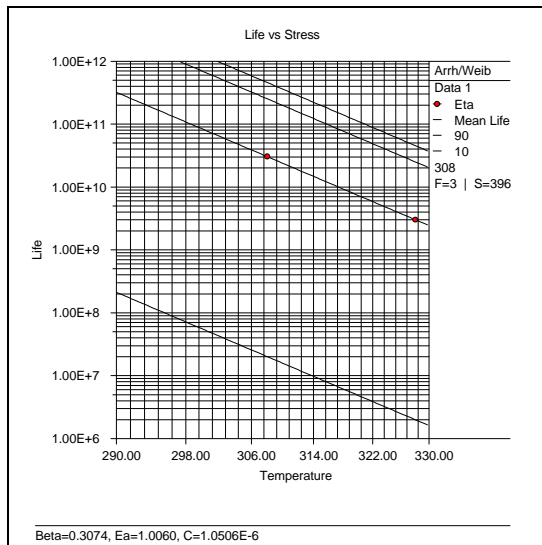


Figure 3: Arrhenius plot for thermally stressed hard drives (use stress level equals 308K)

The mean life at 308K is approximately 2.5264E+11 hours and the mean life at 328K is 2.5048E+10 hours. These values assume no wear mechanisms are present to shorten the life. Therefore,

$$AF = \frac{MTTF_{308}}{MTTF_{328}} = \frac{2.5624E+11}{2.5048E+10} = 10.0863$$

This value equals the one calculated previously. The activation energy is represented by the slope of the lines on the plot. If the acceleration factor were higher, then the slope of the lines would also increase. In addition, any variation in the activation energy will result in a different value for the acceleration factor. For example, if an activation energy of 0.9 was assumed, this would correspond to an acceleration

factor equal to 7.906. The effects are obvious. A 10% change in the value of the activation energy resulted in more than a 20% change in the acceleration factor. Any incorrect assumptions regarding the activation energy could be detrimental to the analysis.

Additional information regarding accelerated life testing and the software package ALTA™ is provided by Mettas [3].

7. CONCLUSIONS

The presented method provides an easier method of estimating the activation energy and acceleration factor using a software application. The resulting analysis can be completed with confidence without introducing the uncertainties that arise when assuming a value. The effect of assuming values cannot be overlooked, especially when the origins of such values leave much to the imagination. Estimating the activation energy and acceleration factor based on the collected accelerated-test data can increase the confidence the engineer has in these values. Since the estimates are based on maximum likelihood theory, it should be pointed out that confidence bounds on the activation energy and acceleration factor can be easily calculated using Fisher Matrix bounds. Information regarding the calculation of confidence bounds using Fisher Matrix bounds can be found in Nelson [1]. In addition, the values estimated are for the entire product that was tested and not just for a particular material. Without a doubt, the estimates obtained are only going to be as good as the data collected. The more information that is contained within the data, the greater the accuracy that can be expected within the estimates. The use of design of experiments can aid in this process. There are many questions that need to be answered before an accelerated test can be undertaken. First of all, what are the covariates that are affecting the life of the product? Which ones do I want to consider? What are the magnitudes of the stress levels at which the test should be conducted? In the end, only a well planned accelerated life test is worth completing.

So while an easier method has been presented for the estimation of the activation energy and acceleration factor, it is but a single step within the development and testing process. Assuming anything at any point during this process can lead to trouble and nothing should be left to chance or the imagination.

8. ACKNOWLEDGMENTS

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