Warranty Calculations For Missiles With Only Current-status Data, Using Bayesian Methods

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SUMMARY AND CONCLUSIONS:

Recent catastrophic failures of the rocket motor on a U.S. missile caused concern in the U.S. Navy that the missile might not be serviceable past its estimated service life of 20 years. This paper analyzes nearly 2000 firings of the motor under field conditions using classical maximum likelihood estimation (MLE) methods and Bayesian methods. MLE methods indicate that there could be less than a 1% chance that the motor will survive past 20 years of life and are not considered credible. Bayesian methods indicate that there is better than a 99% chance that the motors will survive past 20 years of life. We present reasons for preferring the Bayesian analysis and discuss testing schemes for more precise estimates. We comment on the lack of data to perform degradation analysis as a function of temperature cycling for the motor and make recommendations for future missile system data collection.

1. *INTRODUCTION*

U.S. and NATO forces currently field the missile, which we do not identify for proprietary reasons, with approximately 20,000 missiles in the inventory. Production on the current version began in 1982. Since then, there have been roughly 1940 field firings of the missile, occurring at various missile ages. From June 1997 to March 1998 there were three catastrophic failures of the rocket motor during actual missile launches. These failures caused or risked damage to the firing aircraft and tactical mission failure.

Previous to this time, there had been no motor failures during actual firings. This raised the issue of "wear-out" of the rocket motors --- that their chance of failure had increased as a result of aging.

The rocket motor is one of five critical systems that must work for the missile to successfully engage a target. The reliability of the motor is an upper bound on the over-all missile reliability. See Figure 1.

The rocket motor has a dual propellant, with a fast-burning portion for initial acceleration and a slow-burning portion for cruising. The propellant is case bonded to the rocket. Analysts at the Naval Surface Warfare Center at Indianhead, Maryland (NSWC-IH) believe that thermal cycling may be a failure mechanism, causing the bonds between the propellant and motor casing and /or between the fast- and slow-burning propellants to fail. These failed bonds allow much greater ignition of the propellant, leading to explosion. NSWC-IH has attempted to develop a non-destructive inspection (NDI) method to detect these failed bonds, but with limited success to date.

No data has been collected on the thermal cycle exposure of missiles in the field. It is believed to be great, since the missiles may be externally carried on aircraft repeatedly prior to being fired. The temperature difference between surface and tactical elevations can be extreme, and the missile experiences that cycle at least each time the aircraft sorties. The lack of data on sorties or cycles precludes the construction of a cumulative damage model based on that failure mechanism for motor reliability. There may be other aging mechanisms at work.

The only life data available is calendar age, and that is at best a surrogate for the actual aging mechanism. One could stop and declare the problem hopeless, or one can make the admittedly strong assumption that calendar age is proportional to the unknown failure variate and proceed with the analysis. With several billion dollars at stake, an answer dependent on assumptions is better than no answer at all. We proceed with caution, noting the assumption.

Based on the failed motors, missiles older than 13 years have been coded for wartime use only, and all missiles from the lots where missiles failed have been declared unserviceable.

The remainder of this paper discusses the data (section 2), estimates based on maximum likelihood (section 3), estimates based on Bayesian methods (section 4), discussion (section 5), and conclusions and recommendations (section 6).

This paper reports and extends the results of Sorell's master's thesis (Ref. 1) at the Naval Postgraduate School, which Olwell advised.

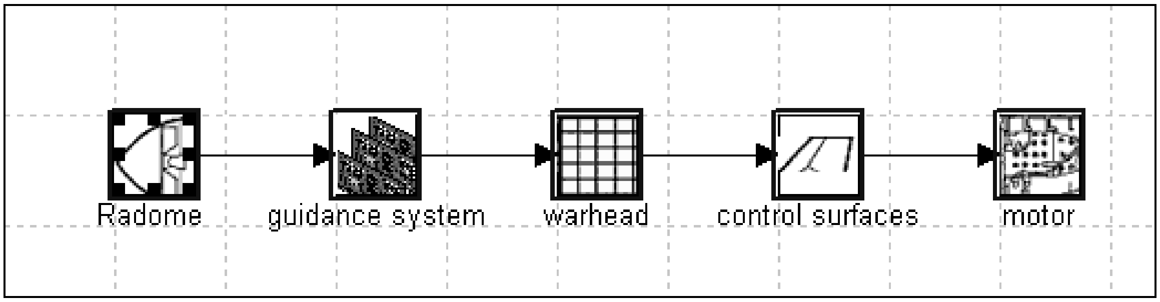

Figure 1. Schematic of missile showing critical onboard systems, including radome, guidance section, warhead, control section and surfaces, and rocket motor.

2. *DATA*

Analysis of the motor life is greatly complicated by the censoring of the failure times. The actual times of wear-out are not known for any motor. For those that were fired successfully, we have suspensions. The motor would have failed at some time past the firing date. For those that failed on firing, we only know that the motor "went bad" some time prior to the firing. This is commonly referred to as left-censored data.

There is not much information in the data. Many of the successful firings occurred at younger ages, and tell us relatively little about the behavior of the more aged missiles. For the missiles that failed, it is impossible to know at what time they changed status from "good" to "bad". Additionally, the scarce number of failures and the relatively few motors fired at older lifetimes present real analytical troubles.

Table one lists summary data on the successful firings. Table two presents summary data on the three motors that failed in actual firings. There have been two motor failures in static tests. One occurred after accelerated testing. A second occurred during exploration of the efficacy of NDI. Neither is included in the data analysis. The first is excluded because of imprecision associated with the acceleration of life times. The second is excluded because it is not known how informative the stresses used in the NDI test are about actual field conditions.

There is no information available on covariates such as number or range of thermal cycles, number of flights prior to firing, number of times handled, shock history, etc. This makes the construction of a cumulative damage model impossible.

3. *MAXIMUM LIKELIHOOD ESTIMATES*

We make the following assumptions in this paper:

1. Missile life can be well modeled by the Weibull distribution. We use the common notation with β being the shape parameter and η being the scale parameter.
2. Missiles for which we have firing data are representative of the remaining population of missiles.

We analyzed the data using two commercially available software packages for life data analysis, and got similar results. The packages were Minitab (Ref. 2) and Weibull++ (Ref. 3). Maximum likelihood analysis for this data is complicated by the fact that there are not sufficient failures for asymptotic normality to be assumed, as indicated in the likelihood plot in Figure 2. Confidence level estimates based on asymptotic normality are likely to be very poorly behaved.

Weibull++ and Minitab both estimate the parameters as

$$\hat{\beta} = 8.1262, \hat{\eta} = 21.229 \qquad (1\text{-}1)$$

and we see the estimated reliability with 95% two-sided confidence bounds in Figure 3. We note that the reliability is estimated to plummet to nearly zero by 27 years. We also know that the estimated value for beta is unusually high.

The implications of these results are that the rocket motor begins to wear out dramatically after 14 years of service, and the reliability drops to nearly zero in ten years. In particular, the time by which one percent of the missiles wears out (B1 time) is estimated to be 12.0 years, with a 90% lower one-sided confidence bound of 10.9 years. Reliability at 20 years is estimated to be 53.9%, with a 90% lower confidence bound of 0.0094%. These results do not accord with engineering belief. MLE estimates for beta tend to be biased high, especially with small samples and heavy censoring and Bain and Englehardt provide correction factors (Ref. 4). This also reduces our acceptance of the MLE estimates for beta.

4. *ESTIMATES BASED ON BAYESIAN METHODS*

When analyzing reliability data with few or no failures, Nelson (Ref. 5) argues to assume a value for the shape parameter and estimate the only scale parameter. Sensitivity analysis by varying the shape parameter indicates how imprecision in the shape parameter affects the results.

Martz and Waller (Ref. 6) point out the lack of a natural conjugate bivariate prior when applying Bayesian methods for the Weibull distribution, and recommend using a discrete prior distribution for the shape parameter for ease of computation. This generalizes Nelson's approach by averaging over discrete

Table 1. Summary data on successful live firings of rocket motor. Note the scarcity of firings of older missiles.

Age at Firing (Yrs)	Last Inspected Time	Quantity Fired	Age at Firing (Yrs)	Last Inspected Time	Quantity Fired
1	0	105	9	0	124
2	0	164	10	0	90
3	0	153	11	0	72
4	0	236	12	0	53
5	0	250	13	0	30
6	0	197	14	0	14
7	0	230	15	0	5
8	0	211	16	0	3

Table 2. Data on missile failures as a result of live firings. It is interesting that all three of the failed missiles were manufactured in early January, although in three different years.

Date of Manufacture	Failure date	Age (years)	Comments
06 Jan 81	27 Jun 97	16.5	Live fire
01 Jan 88	15 Jul 97	8.5	Live fire
09 Jan 84	06 Mar 98	14.2	Live fire

plausible values of the shape parameter, instead of fixing one. They explore using a uniform distribution for the shape parameter. Difficulties with numerical integration argued against using other continuous prior distributions for the shape parameter.

The advent of Markov Chain Monte Carlo methods (Ref. 7) in the last decade allows us to conduct Bayesian analyses without the need to evaluate intractable integrals. We can use any prior distribution, and are not limited to discrete or conjugate priors for ease of computation. This greatly expands the set of possible models for prior distributions. By examining the engineering information available about the missile, we can construct a prior distribution that adds that information into the analysis.

Engineers had expected a much more gradual wear-out of the motor. It is unusual in Weibull analysis to get shape parameters greater than 5. Based on conversations with engineers, we thought that shape parameters with values from one to five were plausible, and we chose to model our prior belief about beta with a gamma distribution with parameters (2, 1). This meant that we expected the shape parameter to be 2, the median value to be 1.67, and that there was only a ten percent chance it was greater than 3.89 and a ten percent chance it was less than 0.53. We used a vague prior for the scale parameter.

Using the usual Bayesian paradigm, we desired to find the predictive distribution for the rest of the motors. Let Y be the life of the next motor. We want the distribution of Y given the previous data. That requires evaluating

$$p(\beta,\eta \mid data) = \frac{p(data \mid \beta,\eta)\, p(\beta,\eta)}{\iint p(data \mid \beta,\eta)\, p(\beta,\eta)\, d\beta\, d\eta} \tag{2-1}$$

and then

$$p(Y \mid data) = \iint f(y \mid \beta,\eta)\, p(\beta,\eta \mid data)\, d\beta\ d\eta\,. \tag{2-2}$$

These integrals are messy, to say the least. Instead of evaluating the integrals directly, we find an approximate solution using Markov Chain Monte Carlo (MCMC) simulation. To do this, we use the WINBugs package, developed at Cambridge, and considered validated by the Bayesian community --- see, for example, the discussion by Carlin and Louis (Ref. 8). With a little work to determine starting points and appropriate burn-in lengths, we obtain the predictive reliability of the missile and a 90% lower probability limit on the reliability. We compare that with the MLE estimates in Figure 4.

We note dramatically different predicted parameters for the missile life. Our prior belief about beta appears to have over-estimated beta. Our posterior median estimate for beta drops to 1.49, while the prior had the median at 1.67. Our 80% posterior interval for beta ranges from 1.14 to 1.68, compared to a prior 80% interval of 0.53 to 3.83.

In particular, we estimate the time by which 1% of the missiles might wear out (median B1 time) as 37.6 years, but there is a 10% chance that the time could be as little as 20.6 years. Expressed another way, we find that there is less than a ten percent chance that as many as 1% of missiles will fail at age twenty years. This is a markedly more optimistic estimate than one obtains with the MLE approach, and is seen as more credible. We note that if we use the posterior mean to estimate the B1 times, the estimated life increases due to the right skew of the posterior distribution.

We were concerned that the prior for beta might be overly influencing the results. We repeated the analysis using a uniform prior for beta over the range of 0.5 to 5.0. Our posterior median for beta dropped to 1.408, with an 80%

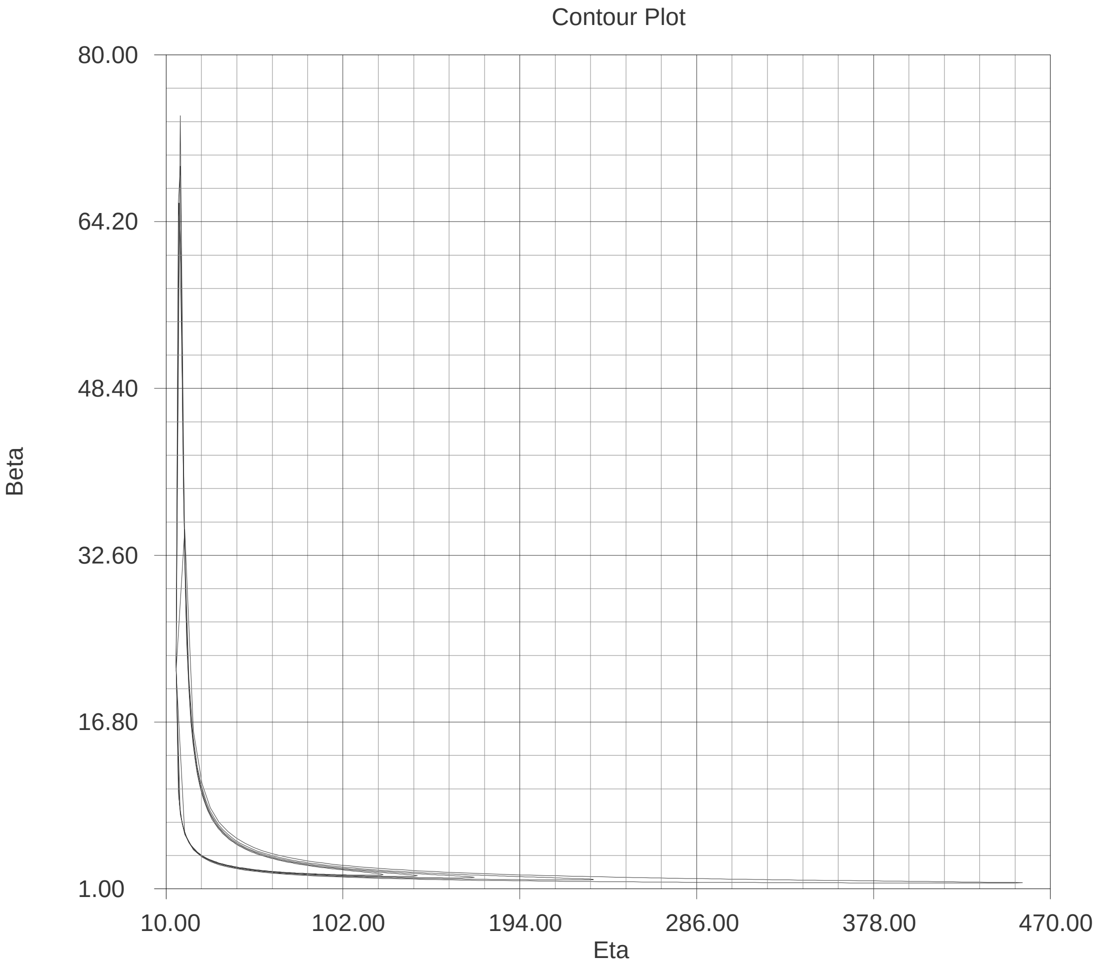

Figure 2. Likelihood contours for beta and eta, at 90%, 80%, 70%, 60%, and 50% relative likelihood. Note the absence of elliptical contours demonstrating the absence of bi-variate normality for the estimators. Also note the wide range of plausible values for both parameters. The scale for beta goes from 0.5 to 70, and the scale for eta from 10 to 450, even at the 50% relative likelihood.

interval of (1.195, 1.5402).

Under this model our B1 time was estimated to be 23.36, with a 10% chance it could be as low as 7.36.

It seems clear that the MLE estimate for beta has an extremely high bias. This argues against using the MLE estimates.

Different simulation runs of the MCMC produce different results, but not markedly different enough to change these conclusions.

4. DISCUSSION

This analysis does not stand or fall with the accuracy of the prior distribution for the parameters, since the results of the uniform prior match those of the informed prior relatively rather closely. The usual frequentist claim is that using prior information distorts the problem analysis. We rebut with two observations. First, if the data did strongly suggest unusually high wear-out indicated by the MLE estimates, we would expect the posterior distribution for beta to have moved much higher, both with the informed and vague priors. This does not occur. Secondly, the engineering judgment supports the lower values, and that information is incorporated into the analysis by the prior for the parameters. Clearly the bias in the MLE estimates is distorting the analysis.

We do retain some concern about the wide difference between the results. Both the MLE models and the Bayesian models depend on the scarce data for missile success at a later life. Since the older missiles are currently suspended from non-wartime use, additional testing of those missiles without exposing aircraft or crews to risk seems warranted. The missiles are not going to be otherwise used. This would allow more information into the analysis, and we know that as the amount of information grows, the Bayesian and MLE estimates will eventually converge.

We also note that these results, both MLE and Bayesian, are dependent on the assumed Weibull distribution. In particular, if there are multiple failure modes that manifest themselves at later life, the Weibull model will not continue to hold. In other words, it is naïve to extrapolate reliability for times too far in the future.

We are particularly fond of Figure 4. It nicely summarizes our best beliefs about the missile reliability, with the 90% lower bound giving a good estimate of our precision in those beliefs, and compares the Bayesian model with the

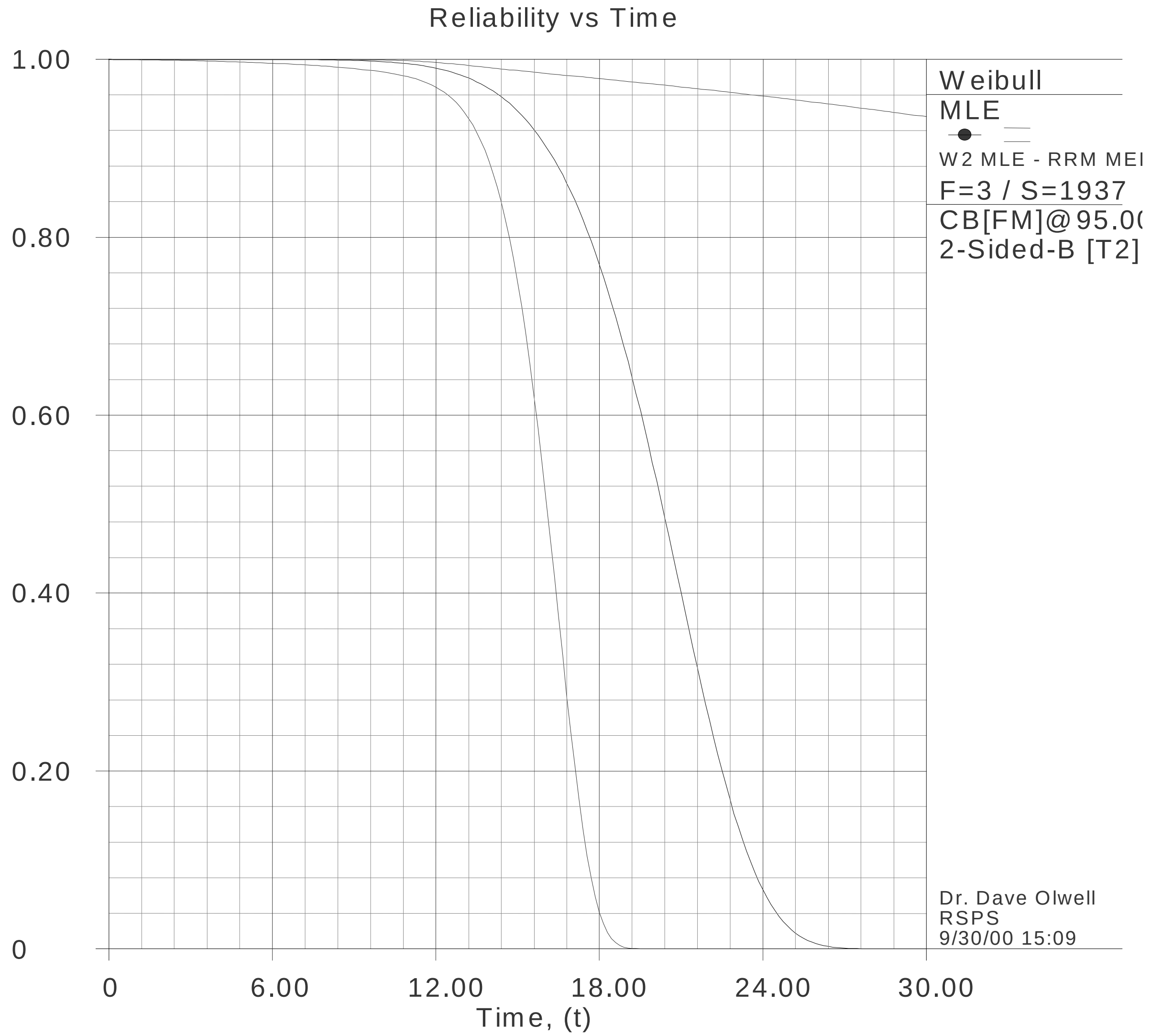

Figure 3 . Reliability versus time for data, generated by Weibull++ using MLE, with 95% two-sided lower confidence bounds . Note the estimated reliability at 25 years is essentially zero, although there is a great deal of uncertainty reflected in the upper bound. The estimated parameters are given as $\beta = 8.13$ and $\eta = 21.23$.

MLE model. This is a graph that a decision maker can easily understand. In particular, if a decision maker can specify an acceptable risk (say, less than a 5% chance of motor failure) Figure 4 can be used to set the service life of the missile under either model.

5. CONCLUSIONS AND RECOMMENDATIONS

We conclude that there is less than a 10% chance that more than 1% of the missiles will fail at twenty years life, based on the information available. This conclusion is sensitive to the prior belief about missile wear out.

We recommend that additional testing of the older, suspended missiles be undertaken to confirm our results. Following confirmation, we recommend that the suspension on missiles older than 13 years be lifted. When the appropriate risk level is determined, a second analysis with all the data should be conducted, a graph like Figure 4 generated, and new missile life determined.

We also recommend that future missile programs include devices to measure the environmental stresses on the missiles, so that cumulative damage models can be constructed. In 20 years, we may be facing the same issues with the PAC3, THAAD, and NMD missile systems, and small expenditures now on “logging” chips (costing under $200) will produce better decisions about these multi-million dollar systems.

6. ACKNOWLEDGEMENTS

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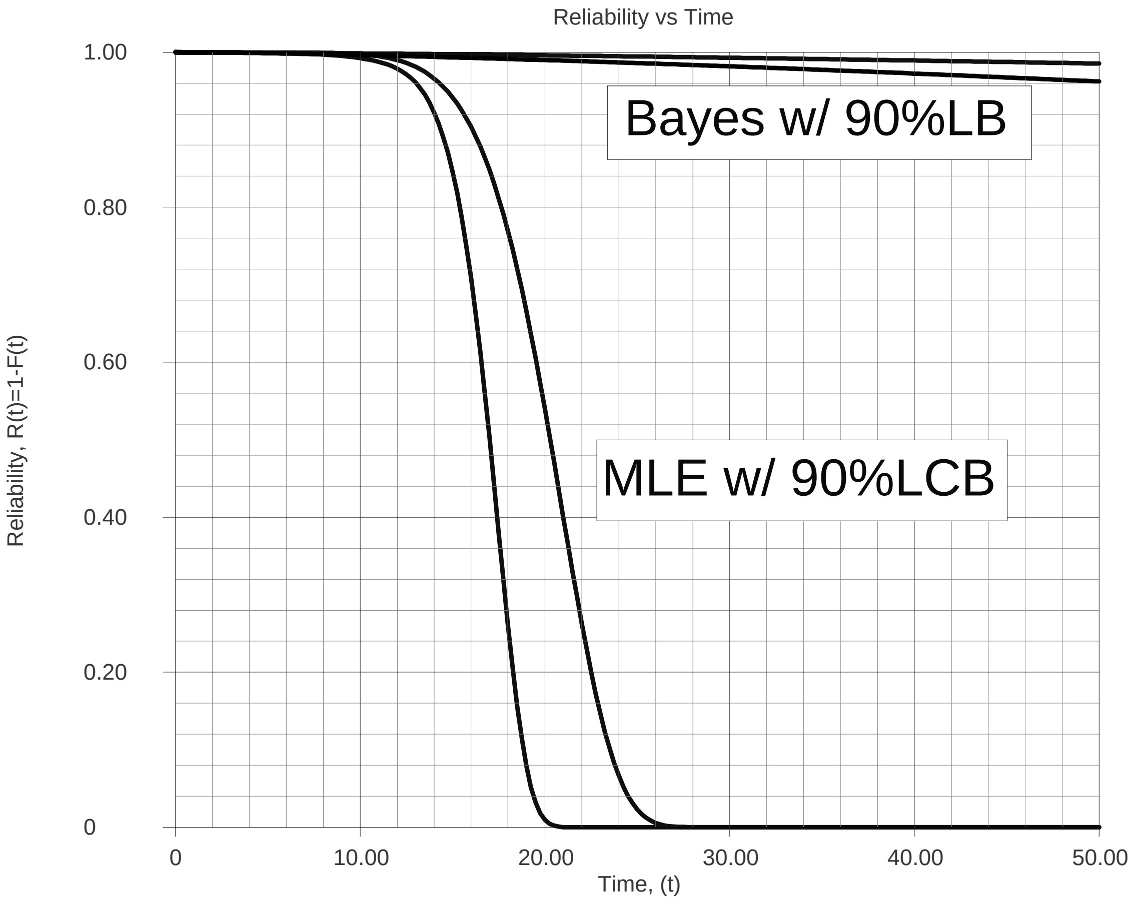

Figure 4. Reliablity versus time for data, using both the Bayesian and MLE analyses. Note the strong drop-off indicated by the MLE model, and the much less gradual decline in reliability for the Bayesian methods with the informed prior. Both have 90% one-sided lower bounds displayed.

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Dr. Olwell holds a PhD in Statistics (1996) and a master's in Mathematics (1989) from the University of Minnesota, and earned a bachelor of science degree from the United States Military Academy at West Point (1980) . He taught mathematics and statistics at West Point for six years, where he also directed the Mathematical Sciences Center of Excellence. He was associate chair of operations research at the Naval Postgraduate School, where he taught reliability, statistics, and quantitative methods. He is the author of several books and numerous papers. His 1998 paper on missile reliability was designated the year’s best paper in air and missile defense by the Military Operations Research Society. Dr. Olwell has taught dozens of seminars on reliability for industry and consults widely for both government and industrial clients. He is the Director of ReliaSoft Professional Services.

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Lieutenant Commander Sorrel recently completed his master’s degree in operational logistics at the Naval Postgraduate School in 2000. He earned a Bachelor of Science degree from the Devry Institute of Technology in 1986 and was commissioned in the United States Navy. A supply officer, he has served on numerous ships during his Navy career. He is currently attending the Supply Officer Department Head course, and somewhat enthusiastically anticipates a return to sea afterwards.