Reliability Predictions based on Customer Usage Stress Profiles

Adamantios Mettas; ReliaSoft Corporation

Key Words: Usage Data, Usage Stress, Time Depended Use Stress, Accelerated Testing, Load Sharing, Warranty Prediction

SUMMARY & CONCLUSIONS

One of the most commonly asked questions by reliability engineers is: "How do I utilize my customer usage data information?" There are different types of customer usage data and each type requires different treatment. This paper presents three types of usage data and their treatment for performing reliability predictions. Accelerated testing data and analysis methods will be utilized (for each type).

1. INTRODUCTION

Accelerated testing is primarily used for shortening test duration. However, the data and the methodologies in Accelerating Testing can be used for a variety of further analyses. In fact, whenever failures are related to stress, these methodologies can provide the means of analysis and prediction for a variety of applications. In other words, an accelerated test can also be described as a life-stress test, which can provide very valuable information regarding the performance of the product in the field under different and changing operating conditions. Clearly, the stress conditions under which a product operates will have a direct effect on its life and reliability.

As an example, consider an aircraft engine, which operates under different stresses during take-off, cruising and landing. The reliability of this engine depends on the time it operates at each stress level. The cumulative damage model [2,4] can be used in such cases to estimate reliability. The first example of this paper reviews a similar type of usage profile for a component that is part of a manufacturing line, and illustrates the use of the cumulative damage model.

On the other hand, the stress conditions depend on the way the product is used and not every customer uses the product in the same way. Certain customers operate the product at higher stress levels than others. For example, every user does not accumulate 12,000 miles a year on a vehicle and every user does not print the same number of pages per week on a printer. The second example in this paper examines such a scenario, where customer usage data were utilized in conjunction with Quantitative Accelerated Life Test (QALT) data in order to perform reliability predictions. Another example of varying stress conditions is the case of load sharing components in a system. Currently, when considering redundant components, it is assumed that the components are statistically independent. In other words, it is assumed that when a component fails the failure distribution of the remaining component(s) is not effected. In most practical applications, however, this assumption is incorrect, since the remaining component(s) have to carry higher load. The third and final example presents this type of analysis. Three software packages [5, 6, 7], will be used to perform the analyses.

2. NOTATION

β	Weibull shape parameter
η	Weibull scale parameter
t_e	equivalent operating time of a unit if it had been operating at a different stress level
t_p	life characteristic
S	stress
x(t)	stress as a function of time
A	parameter of the Eyring relationship
B	parameter of the Arrhenius and Eyring relationship associated with the activation energy
C	parameter of the Arrhenius relationship
K	parameter of the Inverse Power Law relationship
n	parameter of the Inverse Power Law relationship
$f()$	probability density function
$R()$	reliability function
$R^{/}()$	reliability function at an increased stress
IPL	Inverse Power Law
MLE	Maximum Likelihood Estimation method
QALT	Quantitative Accelerated Life Tests

3. THEORY

Accelerated Life models consist of an underlying failure distribution and a Life-Stress relationship. A percentile of the failure distribution (life characteristic, t_p) is represented by the stress-life relationship. Table 1 presents some common Life-Stress relationships.

Relationship	Model
Arrhenius	$t_p = Ae^{\frac{B}{S}}$
Eyring	$t_p = \frac{1}{S}e^{-\left(A-\frac{B}{S}\right)}$
Inverse Power	$t_p = \frac{1}{K \cdot S^n}$

Table 1: Common Life-Stress Relationships

The life characteristic, t_p, can represent any percentile of the distribution. The percentile is selected according to the assumed underline distribution. Some typical life characteristics are presented in Table 2 [2].

Distribution	Parameters	Life Characteristic
Weibull	β, η	Scale Parameter (η)
Exponential	λ	Mean Life ($1/\lambda$)
Lognormal	μ, σ	Median ($\breve{T}$)

Table 2: Typical life characteristics (β and σ are assumed to be constant)

The objective then becomes to obtain the parameters of the failure distribution and the stress-life relationship. For example, the Weibull-IPL model is:

$$f(t,S) = \beta \cdot K \cdot S^n \left(t \cdot K \cdot S^n\right)^{\beta-1} e^{-\left(t \cdot K \cdot S^n\right)^{\beta}}$$

The parameters to be estimated are β, K, and n. In this paper the Maximum Likelihood Estimator (MLE) method is utilized for estimating the model parameters.

4. TIME-DEPENDENT USE STRESS

Consider the data summarized in Table 3. These data illustrate a typical 3-stress level, constant stress accelerated test. The stress in this example is temperature, and the Arrhenius relationship is used.

	Temperature		
	406 K	**416 K**	**426 K**
Time-to-Failure	248	164	92
	456	176	105
	528	289	155
	731	319	184
	813	340	219
		543	235

Table 3: Failure Data

The estimated parameters using MLE are:

β = 2.9658; B = 1.0680E+4; C = 2.3966E-9.

Several types of information about the model as well as the data can be obtained from a probability plot. For example, the choice of an underlying distribution and the assumption of a common slope (shape parameter) can be examined. In this example, the linearity of the data supports the use of the Weibull distribution (**Figure 1**). In addition, the data appear parallel on this plot, therefore reinforcing the assumption of a common beta. Further statistical analysis should be performed for these purposes.

Nelson [4] provides an extensive assessment of model assumptions. For example, a likelihood ratio test can be performed to validate the assumption of a common shape parameter across the three stress levels.

This particular component is a part of a production line, which operates continuously except when the line is shut down during shift changes. Therefore, the use stress as shown in Figure 2 for this product is time-dependent, and is given by:

$$x(t) = \begin{cases} (298 + 116 \cdot t)K, & 0 \le t < 0.5 \ \ hr, \\ 356K, & 0.5 \le t < 8 \ \ hr, \\ (356 - 116 \cdot (t-8))K, & 8 \le t < 8.5 \ \ hr. \end{cases} \quad (1)$$

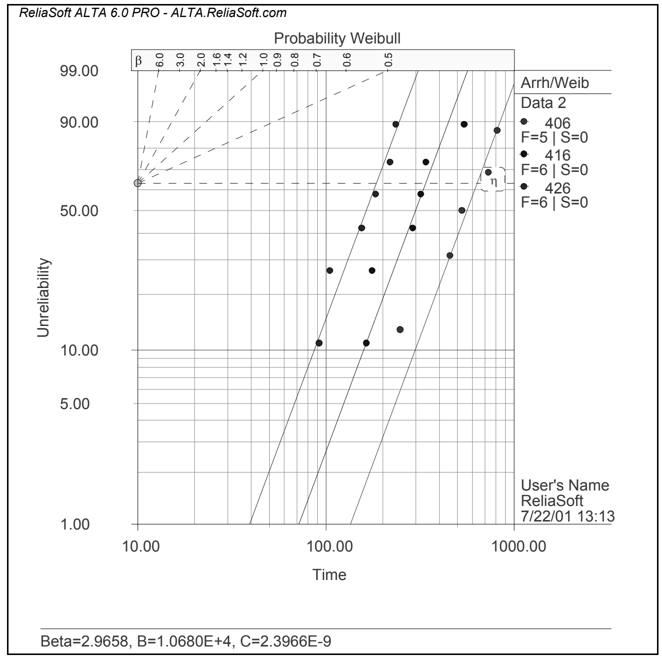

Figure 1: Weibull probability plot of the three test stresses.

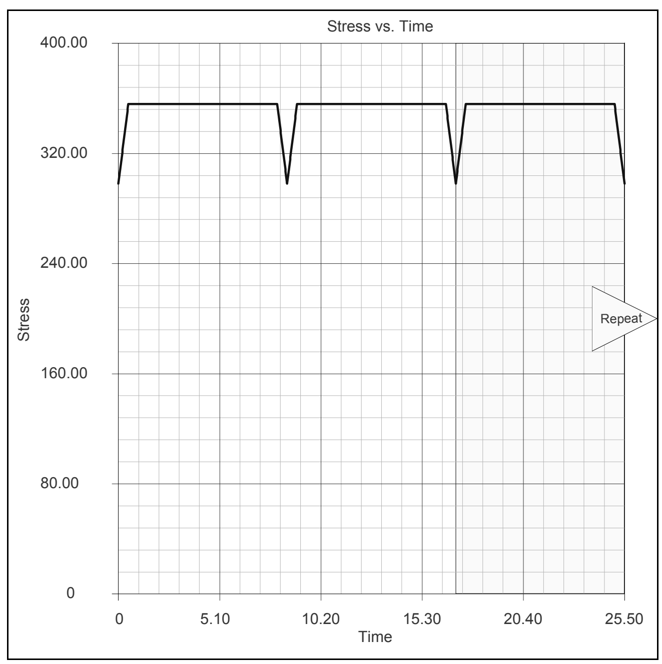

Figure 2: Time-dependent use stress profile.

When the stress is time dependent, the cumulative damage model can be utilized [2], where:

$$R(t, x(t)) = e^{-\left[\int_0^t \frac{1}{C} \cdot e^{-\frac{B}{x(u)}} du\right]^{\beta}} \quad (2)$$

The reliability of this product under the use stress conditions given by equation (1) can be estimated using equation (2). Figure 3 is the reliability plot of this product under the actual operating stress given by equation (2).

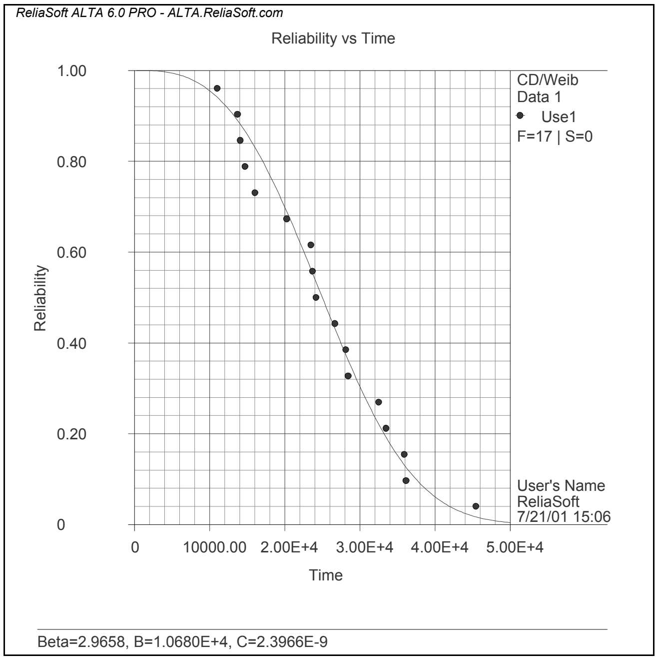

Figure 3: Reliability vs. Time under a time-dependent use stress.

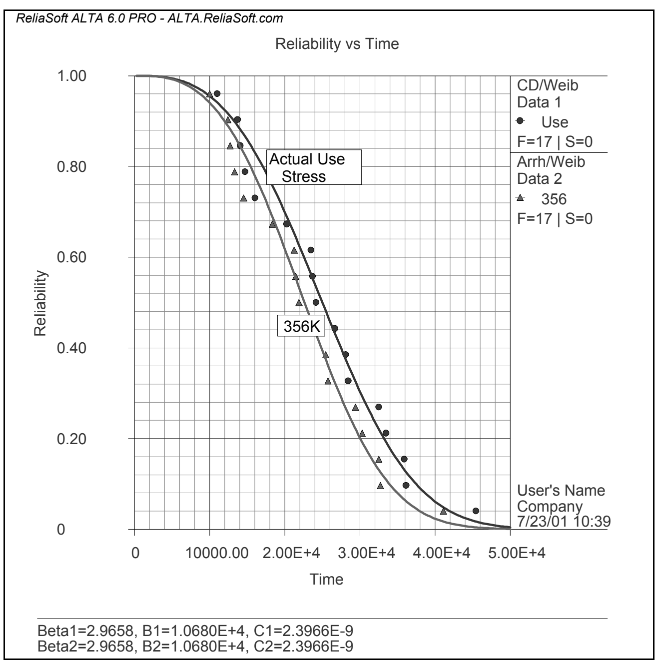

Figure 4: Comparison of the reliability between constant and actual time-dependent operating conditions.

Traditionally, the reliability would have been estimated using a constant stress usage level. For example, in this case it could be assumed that the operating stress is 356K. The reliability for a use stress of 356K is plotted in **Figure 4** next to the reliability under the actual use conditions. The difference between the two reliability curves of **Figure 4** is indicative of the error that can be caused by assuming a constant use stress when in fact the use stress is time-dependent.

5. *PREDICTING RELIABILITY BASED ON CUSTOMER USAGE DATA.*

In recent years, there has been significant research in understanding how the customer is using the product. Many products are designed with built-in devices (such as e-prom chips) that collect data as to how the product is used, the stresses it experiences, the environmental conditions under which it operates, etc. In addition, many companies conduct surveys in order to obtain customer usage information. Despite the significant investment into these efforts, much of this customer usage information is either underutilized, or is not used at all, because the proper analysis method is unknown. This example describes a method for incorporating a type of customer usage profile in reliability predictions, by utilizing accelerated life data.

Consider an electric motor that has been designed for a household appliance. The actual load for the motors in the field will vary depending on the way that the individual user employs each machine. The manufacturer provides a warranty of 1,000 cycles for this motor. In order to make adequate preparations to support the warranty, the manufacturer wants to estimate the percentage of returns that can be expected during the warranty period. The life of the motor is clearly dependent on the applied load and the applied load varies based on customer usage patterns. The question to be answered is which load should be used in predicting the percentage of returns during warranty.

As a first step, the manufacturer decided to obtain information on the life of the motor at different loads, and the data set in **Table 4** was collected. This data set contains

Cycles-to-Failure	Load (lb)
2386	6
3593	6
4045	6
6372	6
6448	6
1414	8
2147	8
3209	8
4026	8
4113	8
5117	8
6127	8
6352	8
819	12
1281	12
1441	12
1796	12
1856	12
2427	12
2645	12
2715	12
3671	12
4881	12

5 motors were suspended at 6500 cycles at the 6 lb load.
1 motor was suspended at 6500 cycles at the 8 lb load.

Table 4**:** Test data for various load sizes

cycles-to-failure information at three different loads (or stress levels) of 6 lbs, 8 lbs and 12 lbs. Five motors were suspended at 6,500 cycles at the 6 lb load. One motor was suspended at 6,500 cycles at the 8 lb load.

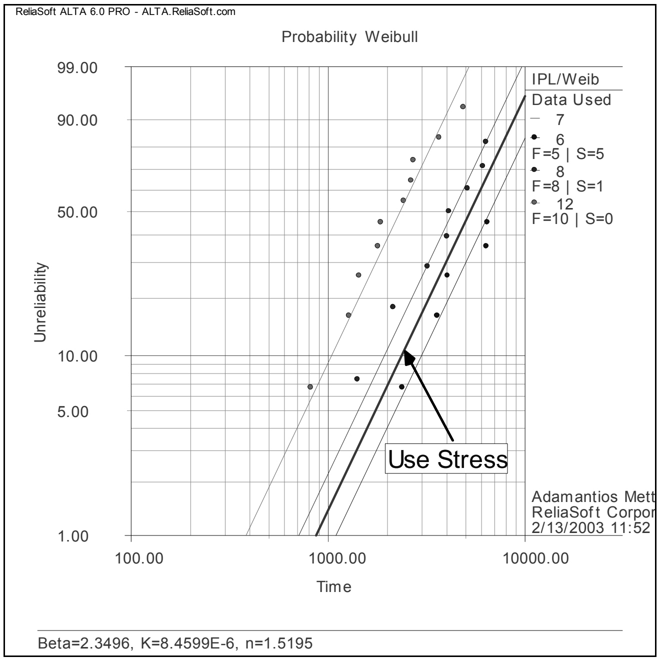

Figure 5: Test and Use stress Weibull probability plot

This data set can be analyzed using accelerated life models. In this case, the Weibull-Inverse Power model was fitted to the data set [6]. The parameters of this model are:

$\beta = 2.3496$, $K = 8.4599\text{E-}6$, and $n = 1.5195$

Figure 5 displays the Weibull probability plot obtained from the analysis. The plot contains a separate line to represent the probability over time for each load (or stress level).

Traditionally, an average usage level would have been used in order to predict the probability of failure for the product within the warranty period. For this motor, it was assumed that 7 lbs is the average use level stress and that 1.39% of the motors will fail within the 1,000 cycle warranty period. However, because customer usage information is also available for this motor, more realistic estimates for the probability of failure for the units in the field can be performed. The customer usage information was obtained by a survey of a representative sample of customers. The distribution that gives the percentage of units at different loads can be determined from this data set. The three parameter Weibull distribution was fitted to the data set and the following parameters were obtained: $\beta = 1.7871$, $\eta = 7.1434$, $\gamma = 1.3649$. Although the data set is too large to be included in this paper, **Figure 6** displays the Probability Density Function (PDF) for this distribution, based on the customer usage information.

The question now is how to relate the load usage distribution to the life of the motor at different loads. The idea is that if we know the fraction of users operating the product at a given stress level and the percentage of units failing at that stress level (for a given time), we can determine the percentage of the population failing at that stress level. For example, consider a population of 200 units. From the stress

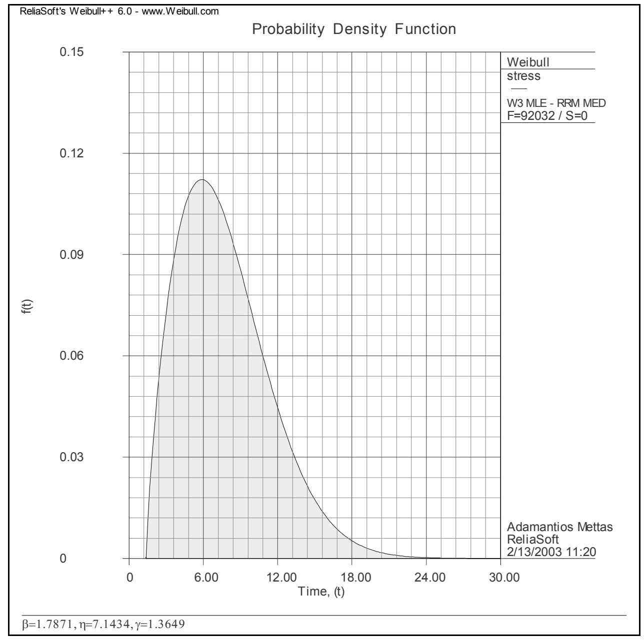

Figure 6: *PDF* from the customer usage data

distribution (usage distribution), we have determined that 10% of these units operate at a stress level *S*. Now we would like to know the probability of failure at *S*. This probability has to be associated with a time as well. If the probability of failure is 30% at time *t* and stress level *S*, then we know that 30% of the 10% of the units that operate at stress level *S* will fail by time *t*, or 6 out of the 200 total units will fail at stress level *S*. This will need to be repeated for all stress levels that are experienced in the field in order to estimate the overall percentage of units failing by time *t*. For this, a stress-strength interference analysis will be used to obtain the percentage failing during warranty from the whole range of load sizes applied in the field. The stress-strength interference equation [3] is given by:

$$P(x_2 \geq x_1) = \int_0^{\infty} f_2(x) \cdot R_1(x) dx \quad (3)$$

Given a distribution that describes the stress levels a unit will experience during its use and a distribution that describes the strength of this unit, *e.g.,* material strength, material properties, design properties, etc.), the probability of failure for this unit can be determined based on the probability of the stress exceeding the strength of the unit as defined in equation (3).

Using this idea, the customer usage information represents the data set required to obtain the stress distribution in equation (3). This distribution describes the percentage of users who operate the product at each stress level. Therefore, if we can obtain the strength distribution, we will be able to use equation (3) to calculate the overall probability of failure of the product, while taking into account the variation in usage patterns among customers. In order to determine the strength distribution, we will need a way to relate life with stress and to estimate the percentage of units failing at each stress level. Utilizing the quantitative accelerated life test data given in **Table 4** and the corresponding analysis, we have already obtained this life-stress relationship and we can now obtain the probability of failure at different stress levels for a given time i.e., the probability of failure at 1,000 cycles for different loads, as presented in **Table 5**.

Load (lb)	% Failing at 1000 Cycles
5	0.4213689387
6	0.8063513205
7	1.393983535
8	2.235892479
9	3.384752347

Table 5: Percentage failing at 1,000 cycles for various loads.

Using this data set, we can obtain the distribution of the percentage of units failing during the warranty period of 1000 cycles at each load size. This can be performed using simple regression. The Weibull distribution was fitted and the following parameters were obtained:

$\beta = 3.57032$ and $\eta = 23.12145$.

This is our strength distribution. We now have two distributions, one giving the percentage of units operating at each load size, and another giving the percentage of units that fail at each load size during the warranty period of 1,000 cycles. **Figure 7** displays the stress distribution compared to the strength distribution.

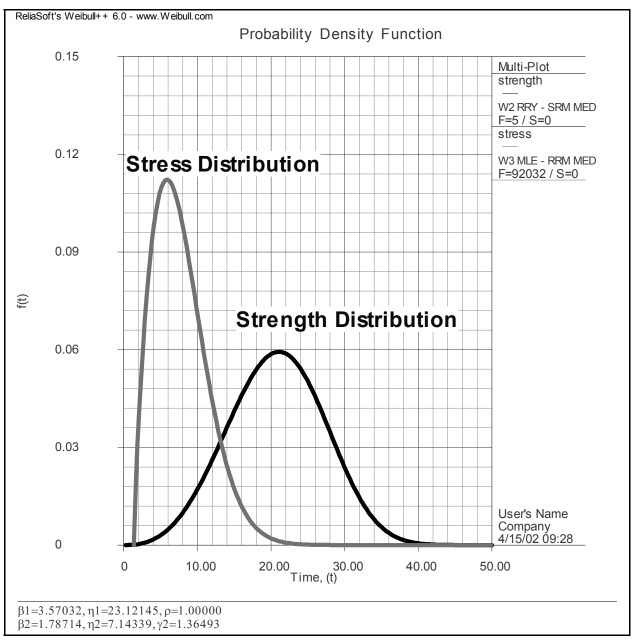

Figure 7: Comparison of stress and strength distributions

Based on this analysis, it is estimated from equation (3), that 4.16% of the units will fail under warranty. This value is significantly higher than the 1.39% obtained by assuming an average load of 7 lbs.

6. LOAD SHARING COMPONENTS

This example examines the application of non-repairable QALT models in system reliability analysis. Consider the case where the reliability of two components in parallel is to be calculated. In most cases, it is assumed that the components are statistically independent. In real applications, however, if one component fails, the component that is still operating will assume the failed unit's load. Therefore, the failure distribution of the surviving unit will change, since it will have to carry 100% of the load. This type of arrangement is called Load Sharing. Kececioglu [1] gives the reliability equation for two load sharing components as follows in equation (4):

$$R(t) = R_1(t) \cdot R_2(t) + \int_0^t f_1(x) \cdot R_2(x) \cdot \frac{R_2'(t_{1e} + (t-x))}{R_2'(t_{1e})}\, dx + \int_0^t f_2(x) \cdot R_1(x) \cdot \frac{R_1'(t_{2e} + (t-x))}{R_1'(t_{2e})}\, dx \tag{4}$$

$R_1'()$ and $R_2'()$ are the reliability functions of each component at the increased stress level. Kececioglu [1] points out that the distributions of each component at the increased stress level can be determined through testing at that stress. This is the same as performing a Quantitative Accelerated Life Test. A distribution and a life-stress relationship can then be fitted to the data, thus obtaining a failure distribution as a function of stress. Therefore, equation (4) can be rewritten in terms of the reliability of each component as a function of stress [3]:

$$R(t) = R_1(t,S_1) \cdot R_2(t,S_2) + \int_0^t f_1(x,S_1) \cdot R_2(x,S_2) \cdot \frac{R_2(t_{1e} + (t-x), S)}{R_2(t_{1e}, S)}\, dx + \int_0^t f_2(x,S_2) \cdot R_1(x,S_1) \cdot \frac{R_1(t_{2e} + (t-x), S)}{R_1(t_{2e}, S)}\, dx \tag{5}$$

where S_1 and S_2 are some portion of the total stress, S, and $R(t,S)$ is the reliability as a function of stress. For example, assuming a Weibull distribution and the Inverse Power Law (IPL) life-stress relationship the reliability of a component is given by:

$$R(t,S) = e^{-\left(t \cdot K \cdot S^n\right)^\beta} \tag{6}$$

Equation (6) is substituted into equation (5), and the reliability of the load sharing system can then be calculated. The benefit of this formulation is that it allows the calculation of the reliability of a load sharing system for different stresses, as well as for different stress allocations for each component. For example, the total stress might be allocated as 70% for component 1 and 30% for component 2 instead of 50% for each component. Also, this formulation can easily be expanded to multiple load sharing components, without requiring additional data or analyses.

Consider a system of two units (power supplies) connected reliability-wise in parallel, which must supply an output voltage of 8 volts. If both units are operational, then each component is to generate 50% of the total output. The purpose of the analysis is to calculate the system reliability at 8,760 hours (1 year). The system is presented in **Figure 8**.

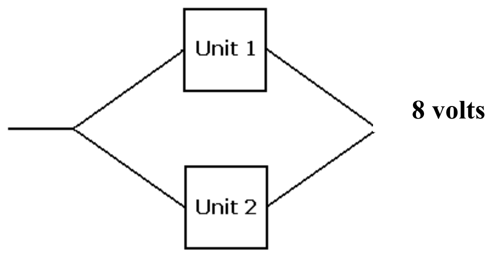

Figure 8: System of two units connected reliability-wise in parallel

If Unit 1 fails, then Unit 2 will assume the responsibility of supplying the entire load by itself. This circumstance will obviously have a great impact on the reliability of Unit 2, since it is now being asked to supply twice the output. This will continue for as long as Unit 1 is down. While this is occurring, the probability of Unit 2 failing due to the increased load (stress) has also increased (compared to when the two units share the load equally). Accelerated Test data were available for this power supply, and are presented in **Table 6**. A total of 20 units were tested to failure at 7, 10, and 15 volt outputs.

	Voltage		
	7 volts	**10 volts**	**15 volts**
Time-to-Failure	874	340	105
	2253	551	167
	3026	560	275
	3115	825	362
	3575	1079	
	3918	1140	
	5000	1701	
	5290	2800	

Table 6: Power Supply Accelerated test data

For this example, Units 1 and 2 are the same part number. Therefore, only one set of data was collected. However, it is possible that the load sharing components may not be the same. If that were the case, data would need to be collected for each component.

The Weibull-IPL model was fitted to the data [6] and the estimated parameters are:

$\beta = 1.9239$, $K = 3.2387\text{E-}7$, and $n = 3.4226$.

The load sharing system's reliability at 8,760 hours can be obtained by substituting these parameters into equations (5) and (6). Solving equation (5), with $S=8$, $S_1=4$, and $S_2=4$, yields [7]:

$$R(8760) = 85.67\%$$

For comparison purposes, if the two power supplies were assumed to be statistically independent, and each to supply 50% of the load, then the reliability of the simple parallel system would be:

$$R(8760) = 1-(1-R_1(8760,4))*(1-R_2(8760,4)) = 98.8\%$$

This result is significantly different than the 85.67% calculated when taking into account the fact that in case of a component failure the remaining component carries 100% of the load.

REFERENCES

1. Kececioglu, D., Reliability Engineering Handbook, Vol. 2, 1991, PTR Prentice Hall.
2. Mettas, A, and Vassiliou, P, "Modeling and Analysis of Time-Dependent Stress Accelerated Life Data," RAMS Proceedings, 2002 Jan
3. Mettas, A, and Vassiliou, P, "Application of Quantitative Accelerated Life Models on Load Sharing Redundancy," RAMS Proceedings, 2004 Jan.
4. Nelson, W., Accelerated Testing: Statistical Models, Test Plans, and Data Analyses, 1990; John Wiley & Sons.
5. ReliaSoft Corporation, Weibull++ Version 6.
6. ReliaSoft Corporation, ALTA™ Version 6.
7. ReliaSoft Corporation, BlockSim™ Version 6.

BIOGRAPHIES

Adamantios Mettas
ReliaSoft Corporation
Corporate R&D
115 S. Sherwood Village Dr.
Tucson, AZ 85710 USA

adamantios.mettas@ReliaSoft.com

Mr. Mettas is the Senior Research Scientist at ReliaSoft Corporation. He fills a critical role in the advancement of ReliaSoft's theoretical research efforts and formulations in the subjects of Life Data Analysis, Accelerated Life Testing, and System Reliability and Maintainability. He has played a key role in the development of ReliaSoft's software including Weibull++, ALTA, and BlockSim, and has published numerous papers on various reliability methods. Mr. Mettas holds a BS Degree in Mechanical Engineering and an MS degree in Reliability Engineering from the University of Arizona.