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*Annual RELIABILITY and MAINTAINABILITY Symposium,*"  
Fort Worth, Texas, USA, January 26-29, 2009.

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# Improving the 1-Parameter Weibull: A Bayesian Approach

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Key Words: 1-parameter Weibull, Bayesian approach, prior distributions, likelihood function

## SUMMARY & CONCLUSIONS

Using maximum likelihood estimation (MLE) to estimate the parameters in a Weibull distribution will lead to a biased estimation of the shape parameter when the sample size is small or too few failures are observed. This bias may lead to inaccurate reliability point estimates. In addition, with few data points available in the calculation, the uncertainty of the estimated parameters is high, which again leads to high uncertainty in the predicted reliability (i.e. wide confidence bounds). To overcome these two issues, the 1-parameter Weibull distribution has been widely used, provided that the shape parameter is known beforehand. This approach, however, does not account for any uncertainty in the assumed value of the shape parameter and can therefore yield optimistic results in the form of tight confidence bounds. It can be improved with better information about the variability of the shaper parameter.

In this paper, a Bayesian model, which is an improved approach for the 1-parameter Weibull, is discussed. Recommendations for establishing variability models for the Weibull shape parameter are presented.

## 1 INTRODUCTION

“We need more samples, we need more samples....” This is the common request of almost every reliability engineer. In order to ensure accurate predictions and statistical inferences, sufficient samples need to be tested and sufficient failures need to be observed. Cost of testing in terms of resources and product cost, as well as multiple projects competing for the available test machines and time-to-market, have pushed sample sizes and test time downward. Obviously, this is not a desirable direction in terms of accuracy, but it is reality.

From a statistical point of view, when dealing with a small sample size or too few observed failures, the uncertainty of the estimates increases, creating wide confidence bounds that verge on being meaningless. To reduce the uncertainty caused by a small sample size, the 1-parameter Weibull distribution has been used [1, 2], especially when prior information about the shape parameter is available. 1-parameter Weibull assumes that the shape parameter of the Weibull distribution is known, which means only the scale

parameter is estimated. Thus, the uncertainty is greatly reduced. However, if the assumed shape parameter of  $\beta$  is far from the true value, the estimated results will be incorrect.

In this paper, a Bayesian model will be discussed. Instead of using a constant value such as in the 1-parameter Weibull, a distribution for the shape parameter of  $\beta$  is considered. The distribution of  $\beta$  can be obtained based on data from developmental testing, prior tests, expert opinions, or field/warranty data. Therefore, this distribution is usually called prior distribution. By using a prior distribution, the uncertainty of the shape parameter is taken into consideration. At the same time, the confidence in the predicted values is improved compared to an analysis in which no prior information is used at all. This method is particularly useful during validation testing, when time is limited and accurate results need to be obtained quickly. In addition, prior tests (developmental tests), which could provide the data for the prior distribution of the shape parameter of the Weibull distribution, have typically been performed prior to the validation stage.

In the following sections, the 1-parameter Weibull model will be discussed first. Then the Bayesian model with discrete and continuous prior distributions will be discussed. A case study is given by applying the Weibull-Bayesian methodology. The results with and without prior information used in the analysis are compared.

## 2 1-PARAMETER WEIBULL MODEL

1-parameter Weibull is, in fact, a special case of a 2-parameter Weibull distribution. The *pdf* of a 2-parameter Weibull distribution is:

$$f(t, \beta, \eta) = \beta/\eta (t/\eta)^{\beta-1} e^{-(t/\eta)^\beta} \quad (1)$$

where:

$f()$ : *pdf* (probability density function).

$\beta$ : The shape parameter.

$\eta$ : The scale parameter.

If the value for  $\beta$  is given (e.g.  $\beta$  is assumed to be 1.5), then the 2-parameter Weibull becomes a 1-parameter model where only  $\eta$  needs to be estimated.

### 2.1 Example

Assume a life test was conducted and the data set is given in Table 1.

Number in State	State F or S	State End Time
1	F	1180
1	F	1842
16	S	2000

Table 1 – Failure Data

There are only two failures in Table 1. If the 2-parameter Weibull model is used, the estimated  $\beta$  is 3.378 and  $\eta$  is 3763.573. The probability plot is given in Figure 1.

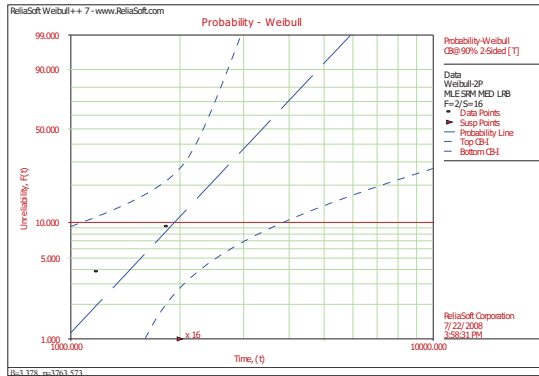


Figure 1 – Probability Plot for 2-Parameter Weibull

The dotted lines are the 90% two-sided bounds on time. Since only two failures are available, the likelihood ratio bounds method, which is preferred over the Fisher bounds when the sample size is small, is used here. These bounds are relatively wide, especially at the two tails. For example, the 90% two-sided bounds for the B10 life are [1073.948, 3856.195] with the predicted B10 life of 1933.236. The ratio of the upper and lower bounds is about 3.59. However, for the B1 life, the bounds are [87.14, 1603.472]. The upper and lower bounds ratio is 18.4, which is quite wide. Given the fact that only 2 failures are available in the calculation, the accuracy of the biased  $\beta$  is also questionable.

Therefore, to reduce the uncertainty, the 1-parameter Weibull model is used and the shape parameter  $\beta$  is assumed to be 1.5 based on historical information. The estimated  $\eta$  is 8439.699. The probability plot with 90% two-sided bounds for time is given in Figure 2.

The 90% two-sided bounds for the B10 life are [982.764, 4914.688] with the predicted B10 life of 1882.69. The upper and lower bounds ratio is 5. From Figure 2, we can see the 1-parameter has an almost constant bounds ratio for any reliability value. To check this, the bounds for the B1 life are calculated. They are [205.172, 1026.040] and the ratio is also 5. Therefore, in general the uncertainty on the predicted reliability is reduced by applying the 1-parameter Weibull model. If the assumed  $\beta$  is correct, then the results will be better than the results from the 2-parameter Weibull.

For comparison purposes, the 1-parameter and 2-parameter Weibull results are plotted together in Figure 3.

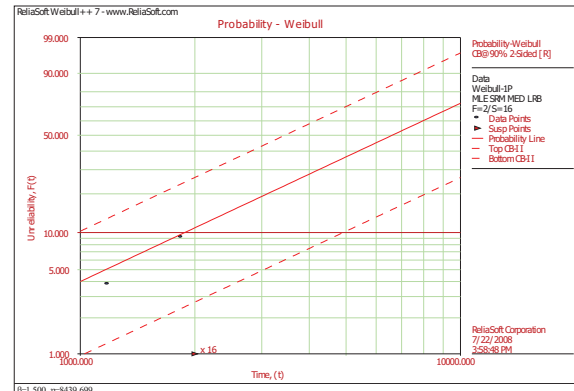


Figure 2 – Probability Plot for 1-Parameter Weibull

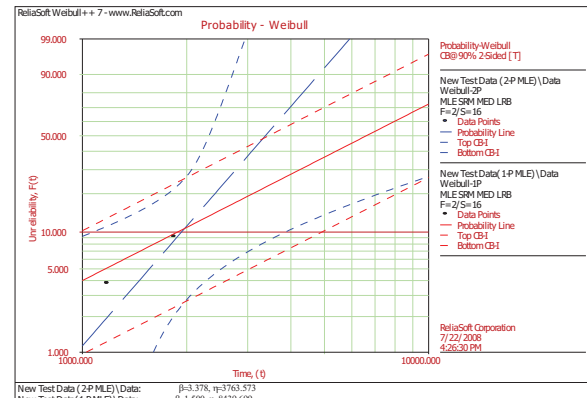


Figure 3 – Probability Plot for 1-Parameter and 2-Parameter Weibull

From the above analysis, it seems that the 1-parameter Weibull model is not a bad option to reduce the model uncertainty and reduce the biasness of  $\beta$ . However, the assumption made in applying 1-parameter Weibull is that the  $\beta$  is known and is correct. In reality,  $\beta$  usually is not known as a single value. It could be several values or a range, or a distribution. How can this prior information on  $\beta$  be used in the calculation? In the following sections, the Bayesian approach, a more realistic approach than the 1-parameter model, is discussed.

### 3 WEIBULL-BAYESIAN MODEL

Bayesian theory is applied in many areas when prior information is available for model parameters. It becomes a natural choice in the reliability engineering field when few failures are observed and prior information is available. For example, Martz, Waller and Fickas [3] used a Bayesian technique to estimate the reliability of series systems of binomial subsystems and components. Willits, Dietz and Moore [4] used a Bayes Monte Carlo approach to estimate the reliability of series systems with very small samples. For many redesigned products, historical test data are available. From these data, engineers might know a rough range for  $\beta$  for a given failure mode. To understand how the Weibull-Bayesian model works, the theory of Bayesian analysis is discussed first in section 3.1.

### 3.1 Bayesian Theory

Bayesian theory is based on Bayes' formula. Assume the occurrence of event  $E$  is affected by a series of mutually exclusive events  $F_i$ . In other words, exactly one of the events  $F_1, F_2, \dots, F_n$  must occur. So the probability of event  $E$  can be obtained by:

$$P(E) = \sum_{i=1}^n P(E|F_i)P(F_i) \quad (2)$$

Suppose now that  $E$  has occurred and we are interested in the probability of which one of the  $F_j$  also occurred. This probability is determined by Bayes' formula:

$$P(F_j|E) = \frac{P(E|F_j)P(F_j)}{\sum_{i=1}^n P(E|F_i)P(F_i)} \quad (3)$$

Equation (3) can be used to improve the 1-parameter Weibull model. Instead of assuming that  $\beta$  is a single value, several values can be used with a predefined probability. For instance, for the example in section 2.1, the prior distribution for  $\beta$  can be assumed to be:

$$P(\beta=3) = 0.3; \quad P(\beta=1.5) = 0.7 \quad (4)$$

That is, the probability for  $\beta=3$  is 0.3 and for  $\beta=1.5$  is 0.7. From this prior distribution, the expected value and variance for  $\beta$  are:

$$\begin{aligned} E(\beta) &= \sum P(\beta = \beta_i)\beta_i = 3 \times 0.3 + 1.5 \times 0.7 \\ &= 1.95 \\ \text{Var}(\beta) &= \sum P(\beta = \beta_i)\beta_i^2 - \bar{\beta}^2 \\ &= 9 \times 0.3 + 2.25 \times 0.7 - 1.95^2 \\ &= 0.4725 \end{aligned}$$

Given the observed data in Table 1, the posterior distribution for  $\beta$  is:

$$P(\beta = \beta_i | \text{data}) = \frac{P(\beta = \beta_i)P(\text{data} | \beta = \beta_i)}{\sum_{i=1}^2 P(\beta = \beta_i)P(\text{data} | \beta = \beta_i)} \quad (5)$$

$P(\text{data} | \beta = \beta_i)$  is the likelihood value for the data set conditioned on  $\beta = \beta_i$ . Applying Equation (5), the posterior distribution for  $\beta = 3$  is:

$$\begin{aligned} P(\beta = 3 | \text{data}) &= \frac{P(\beta = 3) \exp(-20.503)}{P(\beta = 3) \exp(-20.503) + P(\beta = 1.5) \exp(-21.015)} = 0.417 \end{aligned}$$

and for  $\beta = 1.5$  it is:

$$\begin{aligned} P(\beta = 1.5 | \text{data}) &= \frac{P(\beta = 1.5) \exp(-21.015)}{P(\beta = 3) \exp(-20.503) + P(\beta = 1.5) \exp(-21.015)} = 0.583 \end{aligned}$$

The mean and variance for  $\beta$  can be updated from the posterior distribution. They are:

$$E(\beta) = 2.126; \quad \text{Var}(\beta) = 0.545$$

From Equation (5), it can be seen that the 1-parameter Weibull is a special case with  $P(\beta = \beta_0) = 1$ , where  $\beta_0$  is the predefined  $\beta$  value.

In many situations, instead of using a discrete probability like the one given in Equation (4), a continuous distribution such as uniform, normal, lognormal, gamma, etc. is used as the  $\beta$  prior distribution, and it offers a more general approach. In section 3.2, The Weibull-Bayesian model that deals with this general case is discussed.

### 3.2 Weibull-Bayesian Model

The Weibull-Bayesian model, also known as Weibull-Bayesian, is an approach to estimating model parameters using the Bayes technique. In general, if the prior distributions for both the shape and scale parameters are known, Bayes' formula can be written as:

$$g(\beta, \eta | \text{data}) = \frac{P(\text{data} | \beta, \eta) \varphi(\beta, \eta)}{\int_0^\infty \int_0^\infty P(\text{data} | \beta, \eta) \varphi(\beta, \eta) d\beta d\eta} \quad (6)$$

If the prior distributions for  $\beta$  and  $\eta$  are independent, Equation (6) can be modified as:

$$g(\beta, \eta | \text{data}) = \frac{P(\text{data} | \beta, \eta) \varphi(\beta) \varphi(\eta)}{\int_0^\infty \int_0^\infty P(\text{data} | \beta, \eta) \varphi(\beta) \varphi(\eta) d\beta d\eta} \quad (7)$$

where:

$g(\beta, \eta | \text{data})$  is the posterior joint distribution for  $\beta$  and  $\eta$ .

$P(\text{data} | \beta, \eta)$  is the likelihood to get the data for a given  $\beta$  and  $\eta$ .

$\varphi(\beta)$  is the prior distribution for  $\beta$ .

$\varphi(\eta)$  is the prior distribution for  $\eta$ .

From Equation (6) or (7), the posterior marginal distributions for  $\beta$  and  $\eta$  are:

$$\begin{aligned} g(\beta | \text{data}) &= \int_0^\infty g(\beta, \eta | \text{data}) d\eta \\ g(\eta | \text{data}) &= \int_0^\infty g(\beta, \eta | \text{data}) d\beta \end{aligned} \quad (8)$$

In the application of the Weibull-Bayesian model,  $\eta$  is usually assumed to follow a non-informative prior distribution with the density function  $\varphi(\eta) = 1/\eta$ . This is called Jeffrey's prior, and is obtained by performing a logarithmic transformation on  $\eta$ . Specifically, since  $\eta$  is always positive, it can be assumed that  $\ln(\eta)$  follows a uniform distribution,  $U(-\infty, +\infty)$ . A uniform prior distribution implies that no prior information is obtained. Applying Jeffrey's rule [5], which states: "In general, an approximate non-informative prior is taken proportional to the square root of Fisher's information," yields  $\varphi(\eta) = 1/\eta$ . For more detailed discussion on the prior distributions of  $\beta$  and  $\eta$ , please refer to [5, 6].

Once the posterior distribution has been obtained from Equation (7), the *pdf* of the times-to-failure can be calculated by:

$$f(t | \text{data}) = \int_0^\infty \int_0^\infty f(t, \beta, \eta) g(\beta, \eta | \text{data}) d\eta d\beta \quad (9)$$

In the Weibull-Bayesian model, for all the model parameters and reliability matrices such as BX life, reliability at a given time and failure rate at a given time, a distribution (the posterior *pdf*) is obtained, rather than a point estimate as in classical statistics. Therefore, if a point estimate needs to be reported, a point of the posterior *pdf* needs to be calculated. Typical points of the posterior distribution used are the mean (expected value) or median.

#### 3.2.1 Statistical Inference on Model Parameters

The expected value of  $\beta$  is obtained from the posterior distribution by:

$$E(\beta) = \int_0^\infty \int_0^\infty \beta g(\beta, \eta | \text{data}) d\beta d\eta \quad (10)$$

Similarly, the expected value of  $\eta$  is obtained by:

$$E(\eta) = \int_0^\infty \int_0^\infty \eta g(\beta, \eta | data) d\beta d\eta \quad (11)$$

The median points are obtained by solving the following equations for  $\tilde{\beta}$  and  $\tilde{\eta}$  respectively:

$$\int_0^\infty \int_0^{\tilde{\beta}} g(\beta, \eta | data) d\beta d\eta = 0.5 \quad (12)$$

$$\int_0^{\tilde{\eta}} \int_0^\infty g(\beta, \eta | data) d\beta d\eta = 0.5 \quad (13)$$

Similar to Equation (12) and (13), the confidence bounds or, in the Bayes world, the *credible bounds* of the parameters can be calculated. For example, the 10% lower one-sided bound for  $\beta$  is the solution for  $\beta_{10}$  of:

$$\int_0^\infty \int_0^{\beta_{10}} g(\beta, \eta | data) d\beta d\eta = 0.1$$

### 3.2.2 Statistical Inference on Reliability and Time

At a given time  $t$ , the reliability  $R(t)$  is a function of  $\beta$ . The density functions of  $\beta$  and  $R(t)$  have the following relationship:

$$\begin{aligned} f(R_t | data) dR &= g(\beta | data) d\beta \\ &= \left\{ \int_0^\infty g(\beta, \eta | data) d\eta \right\} d\beta \\ &= \left\{ \frac{\int_0^\infty P(data | \beta, \eta) \varphi(\beta) \varphi(\eta) d\eta}{\int_0^\infty \int_0^\infty P(data | \beta, \eta) \varphi(\beta) \varphi(\eta) d\beta d\eta} \right\} d\beta \end{aligned} \quad (14)$$

where:

$f(R_t | data)$  is the posterior *pdf* for  $R(t)$ .

$g(\beta | data)$  is the marginal posterior *pdf* for  $\beta$ .

$$R(t) = R_t = e^{-\left(\frac{t}{\eta}\right)^\beta}$$

The expected value for  $R_t$  is:

$$\begin{aligned} E(R_t) &= \int_0^1 R_t f(R_t | data) dR \\ &= \int_0^\infty \int_0^\infty e^{-\left(\frac{t}{\eta}\right)^\beta} g(\beta, \eta | data) d\beta d\eta \end{aligned} \quad (15)$$

The Bayesian one-sided lower bound for  $R_t$  is:

$$\int_0^{R_L} f(R_t | data) dR = 1 - CL$$

Using Equation (14), it becomes:

$$\frac{\int_0^\infty \int_0^\infty e^{\exp\left(\frac{\ln(-\ln(R_L))}{\beta}\right)} P(data | \beta, \eta) \varphi(\beta) \varphi(\eta) d\eta d\beta}{\int_0^\infty \int_0^\infty P(data | \beta, \eta) \varphi(\beta) \varphi(\eta) d\beta d\eta} = 1 - CL \quad (16)$$

$CL$  is the confidence level. The median and other types of bounds, such as the upper bounds and two-sided bounds, can be obtained in a similar way.

Similarly, the expected time at a given reliability is:

$$\begin{aligned} E(t_R) &= \int_0^\infty t_R f(t_R | data) dt_R \\ &= \int_0^\infty \int_0^\infty \eta \exp\left(\frac{\ln(-\ln(R))}{\beta}\right) g(\beta, \eta | data) d\beta d\eta \end{aligned} \quad (17)$$

The Bayesian one-sided lower bound for  $t_R$  is:

$$\frac{\int_0^\infty \int_0^\infty t_R \exp\left(\frac{\ln(-\ln(R))}{\beta}\right) P(data | \beta, \eta) \varphi(\beta) \varphi(\eta) d\eta d\beta}{\int_0^\infty \int_0^\infty P(data | \beta, \eta) \varphi(\beta) \varphi(\eta) d\beta d\eta} = 1 - CL \quad (18)$$

By treating the reliability matrices, such as the reliability,

time and failure rate, as a function of  $\beta$  and  $\eta$ , the posterior distribution of these matrices can be obtained following the method used in Equation (14). Their expected values and percentiles such as median and bounds can be calculated using methodology similar to that in Equations (15-18).

### 3.3 Example

The Weibull-Bayesian model is applied to the data given in Table 1. A non-informative prior distribution is used for  $\eta$ . Using non-informative prior distribution for  $\eta$  means we do not have any beforehand information on  $\eta$ . The  $\beta$  parameter has historical values given in Table 2.

Betas Obtained for Similar Mode	
	1.3
	1.7
	2.1
	2.4
	3.1
	3.9

Table 2 – Historical Beta Values

A prior distribution for  $\beta$  is determined from the data in Table 2. The prior distribution is found to be a lognormal distribution with  $\mu = 0.8171$  and  $\sigma = 0.4515$ . Using this prior, the posterior distributions for the model parameters and reliability matrices are calculated.

From the posterior distributions for the model parameters, the mean, median and two-sided 90% bounds are given in Table 3.

Parameter	Mean	5%	50% (Median)	95%
beta	2.287	1.087	2.126	4.033
eta	6131.8	3147.35	5934	25753

Table 3 – Parameters from the Posterior Distribution

Percentile	Beta	Percentile	Beta
0.05	1.086764	0.55	2.235563
0.1	1.263209	0.6	2.351947
0.15	1.397261	0.65	2.478063
0.2	1.513274	0.7	2.617674
0.25	1.61997	0.75	2.77642
0.3	1.721756	0.8	2.96354
0.35	1.821408	0.85	3.196086
0.4	1.920933	0.9	3.512073
0.45	2.02203	0.95	4.03267
0.5	2.126	0.99	5.197967

Table 4 – Percentiles for the Posterior Distribution for Beta

The posterior distribution for  $\beta$  can be obtained from Equation (8). However, it is not easy to get the closed form *pdf* by calculating complicated integrals. Numerical algorithms can be used to get the percentiles of the



distribution. Using these percentiles, a suitable distribution can be fitted and used as the posterior distribution for  $\beta$ . The percentiles are calculated and given in Table 4.

It is found that the lognormal distribution with  $\mu = 0.749$  and  $\sigma = 0.3957$  is the best fit for the data in Table 4. Its probability plot is given in Figure 4.

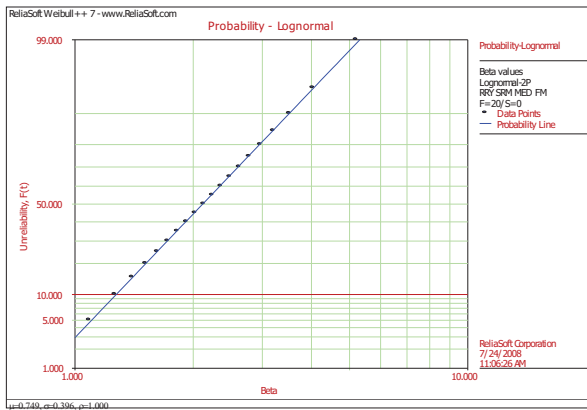


Figure 4 – Probability Plot for the Posterior Distribution for Beta

To compare the prior and posterior distributions for  $\beta$ , their pdf plots are shown in Figure 5.

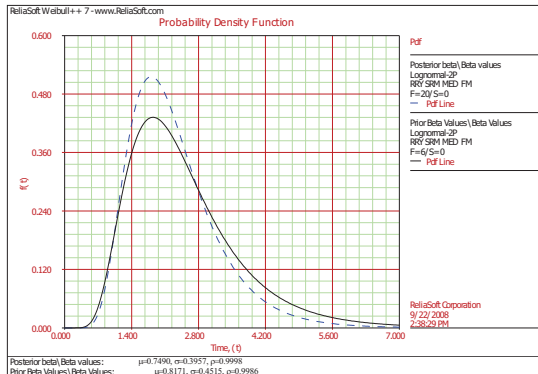


Figure 5 – pdf Plot for the Prior and Posterior Distributions for Beta

The dashed line is the posterior distribution. It can be seen that it is less spread than the original prior distribution.

As mentioned before, at a given time  $t$ , reliability  $R(t)$  is a distribution from the Weibull-Bayesian model. For example, at time 3000, the percentiles of  $R(3000)$  can be calculated numerically. Using these percentiles, a distribution that fits the data well is used as the posterior distribution. This is the same method used to obtain the posterior distribution for  $\beta$ . The probability and pdf plot for  $R(3000)$  are given in Figure 6.

For the data used in this example, if non-informative prior distributions are used for both  $\beta$  and  $\eta$ , a high level of uncertainty will be expected in the results. To compare, the results from using non-informative prior distributions for  $\beta$  and  $\eta$  are plotted together with the results from the above analysis in Figure 7.

In Figure 7, the 90% two-sided bounds are also presented. The solid lines are the results from using the non-informative prior

for beta. The dashed and dotted lines are the results from using the lognormal prior obtained from Table 2. Clearly, using historical information on  $\beta$  provided narrower confidence bounds.

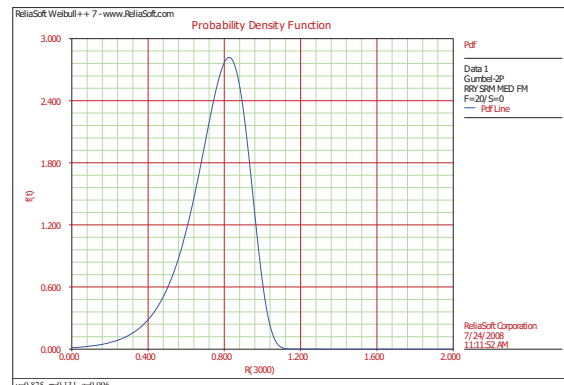
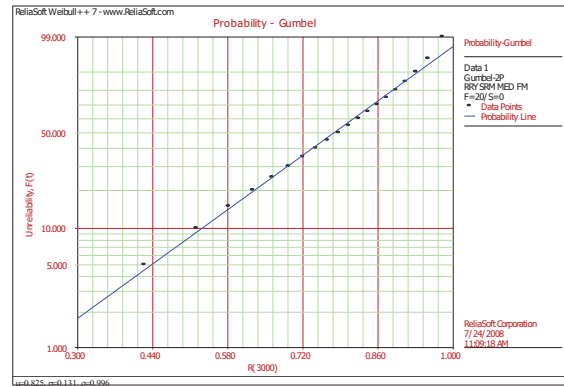


Figure 6 – Probability and pdf Plot for the Posterior Distribution for  $R(3000)$

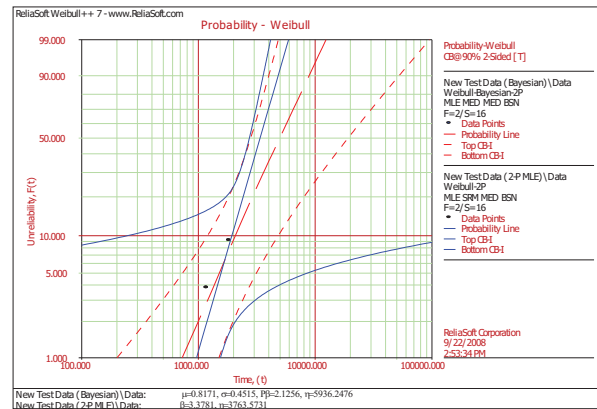


Figure 7 – Probability Plot for Weibull-Bayesian Model with Non-informative and Informative Prior Distributions for Beta

#### 4 CONCLUSION

In this paper, the Weibull-Bayesian model was discussed. An example was provided to illustrate how Bayesian approach can be used to reduce the estimation uncertainty when few failures are observed and history information on  $\beta$  is available. The numerical solutions for the Weibull-Bayesian model were presented. Other techniques to obtain the solutions for the posterior distributions, such as Monte Carlo simulation,

can also be used [7, 8]. The whole premise of Bayesian statistics is to incorporate prior knowledge along with a given set of current observations in order to make statistical inferences. There are many practical applications for this model, particularly when dealing with small sample sizes and some prior knowledge for the shape parameter is available. By incorporating such prior information about a parameter, a posterior distribution for a parameter can be produced and an adequate and more realistic estimate of reliability can be obtained. For example, when a test is performed, there is often a good understanding about the behavior of the failure mode under investigation, primarily through historical data. At the same time, most reliability tests are performed on a limited number of samples. Under these conditions, it would be very useful if this prior knowledge is used with the goal of making more accurate predictions. This approach also reinforces the usefulness and need of setting up and maintaining a database of previous Weibull solutions in order to be prepared for the future use.

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