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Reliability Growth Planning, Analysis and Management

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Summary and purpose

In order to attain the desired reliability objectives early system prototypes are often subjected to reliability growth testing. This testing may be specifically dedicated to reliability or the testing may be integrated into existing engineering development tests. When failure modes are uncovered during the testing they are either corrected or not corrected in accordance with the reliability management strategy. To properly manage this reliability improvement process, the reliability currently achieved and the projected reliability impact of proposed future corrective actions must be appropriately measured. This tutorial presents reliability growth models and procedures to assess reliability growth during development testing and during in-service customer use. Typical development testing and corrective action strategies are discussed and the appropriate models are given for analysis. The tutorial also discusses common analysis pitfalls that lead to incorrect conclusions. The models and methods presented in this tutorial are designed for real world applications and are useful to reliability engineering and program management. These models and concepts are illustrated by numerical examples.

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Dr. Larry H. Crow is president of Crow Reliability Resources, Inc. which provides services and resources in the areas of reliability consulting, training, and software development. Previously Dr. Crow was VP, Reliability & Sustainment Programs, at ALION Science and Technology. From 1985 to 2000 Dr. Crow was Director, Reliability, at General Dynamics ATS—formerly Bell Labs ATS. From 1971-1985, Dr. Crow was chief of the Reliability Methodology Office at the US Army Materiel Systems Analysis Activity (AMSAA). He developed the Crow (AMSAA) reliability growth model, which has been incorporated into US DoD handbooks, and national & international standards. He chaired the tri-service committee to develop Mil-Hdbk-189, *Reliability Growth Management* and is the principal author of that document. He is the principal author of the IEC International Standard 1164, *Reliability Growth-Statistical Tests and Estimation Methods*. Dr. Crow is a Fellow of the American Statistical Association, and the Institute of Environmental Sciences and Technology. He is a Florida State University Alumni Association Distinguished Alumnus and the recipient of the FSU "Grad Made Good" Award for the Year 2000.

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1. INTRODUCTION

In today's environment of compressed schedules and limited testing, every opportunity to identify and correct reliability deficiencies in a new design is a prime objective. A metric for tracking system reliability before development testing based on preemptive corrective actions for potential problem modes is discussed in Ref. 11. In tradition reliability growth testing a system, or subsystem, is tested and failure modes are discovered when the system fails. In accordance with the management strategy some of these failure modes are fixed with a corrective action, while some may not be fixed. If the corrective actions are effective then the reliability will grow to a higher level. In order to manage the reliability growth program we need to measure this reliability growth. We, also, need to be able to evaluate various corrective action options so that priorities can be set as part of the reliability growth management. For further information on the importance of reliability growth as an integration part of a comprehensive reliability plan see Ref. 14.

In addition to managing which failure modes will receive a corrective action other management decisions include when these corrective actions will be incorporated into the system design. These decisions will often influence which parameters are of interest and which model or models are most appropriate for analysis. In the test-fix-test strategy problem modes are found during testing and corrective actions for these problems are incorporated during the test. For the test-find-test strategy problem modes are found during testing but all corrective actions for these problems are delayed and incorporated after the completion of the test. A common approach is a combination of these two approaches, referred to as test-fix-find-test. This is the practical situation where some corrective actions are incorporated during the test and some corrective actions are delayed until the end of the test.

In order to properly manage a reliability growth program it is vital that realistic and valid reliability assessments be made. The correct model and approach is dependent on the corrective action strategy, that is, test-fix-test, test-find-test, or test-fix-find-test. In the test-find-test and the test-fix-find-test situations there are failure modes which have not been corrected at the end of the reliability growth test or the end of current test phase. In these cases there are at least two reliability numbers of interest. One number is the reliability of the system at the end of the test and the other number is the increased reliability when the remaining delayed failure modes are corrected. Both of these numbers are given with the models discussed for test-find-test and the test-fix-find-test.

In Section 2 we give background on the Duane reliability growth postulate, the learning curve growth rate, and the Crow (AMSAA) reliability growth model. In Sections 3, 4, and 5 we discuss data analyses procedures appropriate for the three common development testing and corrective action situations, test-fix-test, test-find-test, and test-fix-find-test. In Section 6 the Extended Continuous Evaluation model is discussed. This model allows for projections on the delayed failure modes and, in addition, allows projection on the failures modes corrected during the testing. In Section 7 we discuss reliability growth assessments and models appropriate for discrete, one-shot systems, such as missiles. In Section 8 we present historical data for key reliability growth parameters and in Section 9 we discuss reliability growth planning. These methods are illustrated by numerical examples.

The overall objective of reliability growth testing is to develop a system with improved reliability for the customer. The impact of reliability on the cost of maintaining a fleet of repairable systems is obvious and for methods and further discussion on this topic see Ref. 12.

1.1 Notation

λ	Scale parameter for Crow (AMSAA) model
β	Shape parameter for Crow (AMSAA) model
α	Growth rate for Crow (AMSAA) model
t	Test time
T	Total test time
$r(\cdot)$	Crow (AMSAA) model failure intensity
X_i	The i-th successive failure time
N	Total number of failures
N_q	Number of failures for q-th interval
λ_A	Type A modes failure intensity
λ_B	Type B modes failure intensity
λ_P	Projected failure intensity
M_P	Projected MTBF
$h(\cdot)$	Rate of uncovering new failure modes
λ_{GP}	Growth potential failure intensity
M_{GP}	Growth potential MTBF
λ_{EM}	Extended model failure intensity
M_{EM}	Extended model MTBF
β_D	Rate of Discovery Parameter
d	Average Fix Effectiveness Factor

2. BACKGROUND on THE DUANE POSTULATE, GROWTH RATES and THE CROW (AMSAA) MODEL

In Section 3 the Crow (AMSAA) model is discussed for the analysis of test-fix-test data. The Crow (AMSAA) model has the same reliability growth pattern as the Duane postulate. This an empirically based learning curve pattern as first noted by Duane and is the most common reliability growth pattern experienced in practice. In this section we give some background on Duane postulate, the related learning curve growth rate, and the Crow (AMSAA) model.

2.1. *The Duane Postulate*

In 1962 J. T. Duane of General Electric published a paper, Ref.1, in which he plotted data from several system undergoing reliability growth testing at General Electric. For each system Duane plotted the cumulative or average failure rate (intensity) at various time points during the testing. If N(t) is the cumulative number of failures up to time t during the testing Duane calculated the cumulative failure rate given by

$$C(t) = \frac{N(t)}{t} \tag{1}$$

Duane then plotted $\ln(C(t))$ versus $\ln(t)$. See Figure 1. The reason Duane plotted this data on $\ln-\ln$ scale is because if these plots are linear then this is consistent with characteristics of a learning curve. With learning most progress is made early then the return as a function of time decreases. With reliability growth most progress is also early due to seeing and correcting the early "valid few" failure modes and later the increase in reliability is slower. As observed by Duane, Table 1, the plots are close to being linear when plotted on $\ln-\ln$ scale. From this relationship Duane noted the parameter "Growth Rate" in his paper.

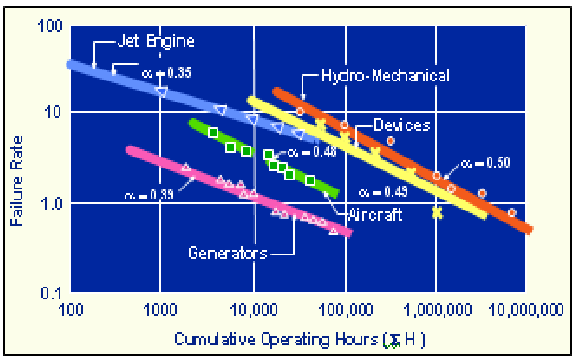

Figure 1. Original Duane Data

In Duane's paper he made this following observations based on the data in Table 1. If the cumulative failure rate $C(t)$ is linear and decreasing on $\ln-\ln$ scale then this implies that

$$\ln\left[C(t)\right] = \delta - \alpha \ln(t) \tag{2}$$

where δ and α are positive parameters.

Equation 2 implies that

$$C(t) = \lambda t^{-\alpha} \tag{3}$$

where $\lambda = e^{\delta}$.

It then follows from (1) that

$$N(t) = \lambda t^{(1-\alpha)}. \tag{4}$$

Next Duane noted that the instantaneous failure rate $r(t)$ (intensity) at time t is the rate of change in the cumulative number of failures $N(t)$. That is,

$$r(t) = \frac{d}{dt}\left[N(t)\right] = \lambda(1-\alpha)t^{-\alpha}. \tag{5}$$

Following Duane, the instantaneous Mean Time Between Failure is

$$M(t) = \frac{1}{r(t)}.$$

Duane called the parameter α the "Growth Rate." This is the Duane Postulate, and is empirically based. The Duane Postulate is deterministic in the sense that there is no underlying statistical framework or foundation for the reliability growth data

2.2. Learning Curve Growth Rate Property

The linear pattern on $\ln-\ln$ scale plotted by Duane is the most common pattern empirically observed during reliability growth testing. This pattern reflects a learning curve characteristic which typically should be expected during reliability growth. Specifically, reliability growth takes place as the instantaneous failure intensity becomes less than the average failure intensity. As noted above, the cumulative failure intensity is $C(t) = \lambda t^{-\alpha}$ and the instantaneous failure intensity is

$$r(t) = \lambda(1-\alpha)t^{-\alpha}. \tag{6}$$

The difference is

$$D(t) = \lambda t^{-\alpha} - \lambda(1-\alpha)t^{-\alpha} = \lambda t^{-\alpha}\alpha \text{ and}$$

$$\ln(D(t)) = \text{Constant} + \alpha \ln(t). \tag{7}$$

Therefore, the Growth Rate α is the rate on $\ln-\ln$ scale in which the instantaneous failure intensity is less than the cumulative failure intensity. All the models discussed in this paper have this learning curve Growth Rate property.

2.3. Crow(AMSAA) Model

The Duane postulate, Ref.1, for reliability growth states that the instantaneous system MTBF at cumulative test time t is $M(t) = \left[\lambda\beta t^{\beta-1}\right]^{-1}$, where $0 < \lambda$ and $0 < \beta$ are parameters.

Crow, Refs. 2, 3, modeled the Duane postulate stochastically as a non-homogeneous Poisson process (NHPP) with intensity

$$r(t) = \lambda\beta t^{\beta-1}, \tag{8}$$

thus allowing for statistical procedures based on this process for reliability growth analyses. This model is applicable to test-fix-test data.

The parameter λ is referred to as the scale parameter and β is the shape parameter. For $\beta = 1$, there is no reliability growth. For $\beta < 1$, there is positive reliability growth. That is, the system reliability is improving due to corrective actions. For $\beta > 1$, there is negative reliability growth.

Under the Crow (AMSAA) basic model the achieved or demonstrated failure intensity at time T, the end of the test, is given by r(T). For the Crow (AMSAA) model we denote the achieved failure intensity by

$$r_{CA} = r(T). \tag{9}$$

The achieved or demonstrated MTBF at time T is given by MT) =1/r(T).

During the testing failures occur which are caused by the corresponding failure modes. A repair restores the system to an operating status, but the reliability has not been improved. A fix or corrective action is aimed at improving the reliability of the failure mode to reduce its rate of occurrence. Management makes a decision to either continue to repair a failure mode (no corrective action) or to implement a fix. It may take time to implement a corrective action so the failure mode may be repaired one or more times before a corrective action is incorporated into the system. The test-fix-test strategy is to incorporate all corrective action into the system during the testing.

During testing the actual failure times may be known. In some practical applications only the number of failures over intervals of time may be known and available for analysis. This situation is called "grouped data." These two cases are covered next.

3. RELIABILITY GROWTH MODELS AND ESTIMATION PROCEDURES for TEST-FIX-TEST DATA.

In this section models and procedures are given for analyzing reliability growth for test-fix-test data.

3.1. Test-Fix-Test with Failure Times Known

Suppose a development testing program begins at time 0 and is conducted until time T and stopped. Let N be the total number of failures recorded and let $0<X_1 < X_2 < \ldots X_N < T$ denote the known N successive failure times on a cumulative time scale. We assume that the Crow (AMSAA) NHPP assumption applies to this set of data. Under the Crow (AMSAA) model the maximum likelihood estimates (MLEs) for λ and β (numerator of MLE for β adjusted from N to N-1 to obtain unbiased estimate) are (see also Refs. 4, 10, 15)

$$\hat{\lambda} = \frac{N}{T^{\hat{\beta}}} \quad , \quad \hat{\beta} = \frac{N-1}{\sum_{i=1}^{N} \ln\left(\frac{T}{X_i}\right)} \quad . \tag{10}$$

Example 1. Test-Fix-Test with Failure Times Known

To illustrate the general application of this model consider a system tested for T=400 hours with the 56 failure times given in Table 1. The first failure was recorded at .7 hours into the test, the second failure was recorded 3 hours later at 3.7. The last failure occurred at 395.2 hours into the test, and the testing was stopped 4.8 hours later at 400.

0.7	63.6	125.5	244.8	315.4	*366.3*
3.7	72.2	133.4	249	317.1	*373*
13.2	99.2	151	250.8	320.6	*379.4*
15	99.6	163	260.1	324.5	*389*
17.6	100.3	164.7	263.5	324.9	*394.9*
25.3	102.5	174.5	273.1	342	*395.2*
47.5	112	177.4	274.7	350.2	
54	112.2	191.6	282.8	355.2	
54.5	120.9	192.7	285	364.6	
56.4	121.9	*213*	*304*	*364.9*	

Table 1. Test-Fix-Test Data

Applying equations (10) we get the estimates

$$\hat{\lambda} = 0.2397 \quad \hat{\beta} = 0.9103 \ . \tag{11}$$

The achieved or demonstrated failure intensity and MTBF are estimated by

$$\hat{\lambda}_{CA} = \hat{\lambda}\hat{\beta}T^{\hat{\beta}-1} \text{ or } \hat{\lambda}_{CA} = 0.1274 \ , \tag{12}$$

$$\hat{M}_{CA} = \left[\hat{\lambda}_{CA}\right]^{-1} = 7.84. \tag{13}$$

It is important to note that the Crow (AMSAA) test-fix test model does not assume that all failures in the data set (e.g. Table 1) receive a corrective action. Based on the management strategy some failures may receive a corrective action and some may not. This is discussed further in Sections 4, 5, and 6.

3. 2. Test-Fix-Test with Grouped Data

This is the grouped data version of the Crow (AMSAA) model that addresses the situation where the actual failure times may not be known. In this case the total test period is partition into K intervals and the number of failures in each interval is known. It is not required that the intervals be of the same length.

Let the length of the q th interval be L_q , q = 1,…,K. Also, let $T_1 = L_1$, $T_2 = L_1 + L_2$, …, etc, be the accumulated time through the q-th interval. Let N_q be the total number of failures in the q-th interval. See Table 2.

Interval	No.of Failures	Length	Accum. Time
1	N_1	L_1	T_1
2	N_2	L_2	T_2
q	N_q	L_q	TS_q
K	N_K	L_K	T_K

Table 2: Grouped Data for Test-Fix-Test

The Crow(AMSAA) model failure intensity is estimated by

$$\hat{r}(T) = \hat{\lambda}\hat{\beta}T^{\hat{\beta}-1} \tag{14}$$

where the values $\hat{\lambda}$ and $\hat{\beta}$ satisfy

$$\sum_{q=1}^{K} N_q \left[\frac{\left\{ \left[S_q\right]^{\hat{\beta}} Ln\left[S_q\right] - \left[S_{q-1}\right]^{\hat{\beta}} Ln\left[S_{q-1}\right] \right\}}{\left[S_q\right]^{\hat{\beta}} - \left[S_{q-1}\right]^{\hat{\beta}}} \right] \tag{15}$$

$$\hat{\lambda} = \frac{N}{T^{\hat{\beta}}} . \tag{16}$$

where $N = N_1 + N_2 \ldots + N_q$ is the total number of failures. The achieved or demonstrated MTBF is estimated by

$$\hat{M}_{CA} = \left[\hat{\lambda}_{CA}\right]^{-1} . \tag{17}$$

Example 2. Test-Fix-Test with Grouped data

Consider the data in Table 3 where the total test time is 500 hours. The 500 hour test time is partition into 6 intervals and the total number of failures over each interval is known. That is, K = 6. See Table 3.

Interval	No.of Failures N_q	Interval Length L_q	Accum. Time T_q
1	12	62	62
2	6	38	100
3	15	87	187
4	3	23	210
5	18	140	350
6	16	150	500

Table 3. Grouped Data

$$\hat{\lambda} = 0.446 \text{ and } \hat{\beta} = 0.814. \quad (18)$$

This gives the achieved failure intensity

$$\hat{r}(T) = \hat{\lambda}\hat{\beta}T^{\hat{\beta}-1} = 0.114. \quad (19)$$

and the achieved MTBF

$$\hat{M}(T) = \left(\hat{\lambda}\hat{\beta}T^{\hat{\beta}-1}\right)^{-1} = 8.7\,. \quad (20)$$

4.0. Projection Model for Test-Find-Test

Suppose a system is tested for time T. During the testing problem failure modes are identified, but all corrected actions are delayed and incorporated at the end of the test phase. This is test-find-test. These delayed corrective actions are usually incorporated as a group and the result is generally a distinct jump in the system reliability. The projection model developed by Crow in 1983, Ref. 5, estimates this jump in reliability due to the delayed fixes. This is called a "projection."

The projection model places all failure into two groups, A and B. Type A failure modes are all modes such that if seen during test no corrective action will be taken. This accounts for all modes for which management determines that it is not cost-effective to increase the reliability by a design change. Type B failure modes are all modes such that if seen during test a corrective action will be taken. This Type A and Type B determination helps define the reliability growth management strategy. The basic projection model assumes that the Type A failure modes has constant failure intensity λ_A, the i-th Type B failure mode follows the exponential distribution with failure rate λ_i, and the initial failure intensity for Type B failure modes is λ_B.

Example 3. Test-Find-Test

For the data in Table 4 the system is tested for T=400 hours. There is a total of N=42 failures and all corrective actions will be incorporated at the end of the 400 hour test. Each failure is designated as either a Type A failure mode (no corrective action) or Type B failure mode (corrective action). There are $N_A = 10$ Type A failures and $N_B = 32$ Type B failures during the test.

In Table 4 there are M = 16 unique Type B failure modes seen which means there are 16 distinct corrective actions incorporated into the system at the end of test. The total number of failures for the j-th observed distinct Type B mode is denoted by N_j and the total number of Type B failures seen during the test is $N_B = \sum_{j=1}^{M} N_j$. See Table 5.

j	X_j	Mode	j	X_j	Mode
1	15	B1	22	260.1	B1
2	25.3	B2	23	263.5	B8
3	47.5	B3	24	273.1	A
4	54	B4	25	274.7	B6
5	56.4	B5	26	285	B13
6	63.6	A	27	304	B9
7	72.2	B5	28	315.4	B4
8	99.6	B6	29	317.1	A
9	100.3	B7	30	320.6	A
10	102.5	A	31	324.5	B12
11	112	B8	32	324.9	B10
12	120.9	B2	33	342	B5
13	125.5	B9	34	350.2	B3
14	133.4	B10	35	364.6	B10
15	164.7	B9	36	364.9	A
16	177.4	B10	37	366.3	B2
17	192.7	B11	38	373	B8
18	213	A	39	379.4	B14
19	244.8	A	40	389	B15
20	249	B12	41	394.9	A
21	250.8	A	42	395.2	B16

Table 4: Test-Find-Test Data

B Mode j	Number N_j	First Occurrence	EF d_j
1	2	15.0	.67
2	3	25.3	.72
3	2	47.5	.77
4	2	54.0	.77
5	3	56.4	.87
6	2	99.6	.92
7	1	100.3	.50
8	3	112.0	.85
9	3	125.5	.89
10	4	133.4	.74
11	1	192.7	.70
12	2	249.0	.63
13	1	285.0	.64
14	1	379.4	.72
15	1	389.0	.69
16	1	395.2	.46

Table 5: Test-Find-Test Type B Failure Mode Data and Effectiveness Factors

An effectiveness factor (EF) d_j is the fraction decrease in λ_j after a corrective action has been made for the j-th Type B mode. The failure rate for the i-th Type B failure mode after a corrective action is (1-d_j)λ_j. In practice, for application of the projection model, the EFs are assigned based on engineering assessments, test results, etc. Studies indicate that an average EF d of about .70 is typical for a reliability growth program. Individual EFs may vary. The assigned EFs for distinct Type B modes are given in Table 5.

For test-find-test the system failure intensity is constant, say, λ_S, during the testing and then jumps to a lower value due to the incorporation of corrective actions. The intensity at the end of the test T, before delayed corrective actions are introduced into the system, is the achieved intensity. The reciprocal of the intensity is the achieved MTBF M_S.

We estimate the achieved failure intensity λ_S by

$$\hat{\lambda}_S = \hat{\lambda}_A + \hat{\lambda}_B, \quad \hat{\lambda}_A = NA/T, \quad \hat{\lambda}_B = N_B/T. \tag{21}$$

Based on the data in Table 4,

$$\hat{\lambda}_S = 0.105 \quad \hat{\lambda}_A = 0.025, \quad \hat{\lambda}_B = 0.08. \tag{22}$$

The estimated achieved MTBF M_S at time T = 400 before the jump is $\hat{M}_S = 9.5$. We will estimate the jump next.

The estimated projected failure intensity, Ref. 5, is

$$\hat{\lambda}_P = \hat{\lambda}_A + \sum_{j=1}^{M}(1-d_j)\frac{N_j}{T} + \bar{d}\,\hat{h}(T) \tag{23}$$

where $$\bar{d} = \frac{\sum_{j=1}^{M} d_j}{M},$$

is the average EF, and

$$\hat{h}(T) = \hat{\lambda}\,\hat{\beta}\,T^{\hat{\beta}-1}. \tag{24}$$

The estimated projected MTBF is

$$\hat{M}_P(T) = \left(\hat{\lambda}_P\right)^{-1} \tag{25}$$

The projection model $\hat{\lambda}$ and $\hat{\beta}$ for (24) use only the M first occurrence failure times of the seen and unique Type B failure modes. These first occurrences are given in Table 5. Applying equations (10) to the first occurrence data in Table 5 we have

$$\hat{\lambda} = 0.1820, \quad \hat{\beta} = 0.7472. \tag{26}$$

Based on the data in Tables 4 and 5, we have

$$M = 16, \quad \bar{d} = .72, \quad \hat{h}(400) = 0.0299. \tag{27}$$

Also,

$$\bar{d}\,\hat{h}(T) = 0.0215, \quad \hat{\lambda}_A = 0.025$$
$$\sum_{j=1}^{M}(1-d_j)\frac{N_J}{T} = 0.0196. \tag{28}$$

From (9) the projected failure intensity and MTBF are

$$\hat{\lambda}_p = 0.066, \qquad \hat{M}_p = 15.1. \tag{29}$$

The projection model estimates that the MTBF jumps to 15.1 hours from 9.5 hours due to the 16 distinct corrective actions.

5.0. Extended Reliability Growth Model for Test-Fix-Find-Test

This model, Crow, (Ref.13), extends the previous projection model, discussed in Section 4, to include test-fix-find-test data. That is, the 1983 projection model is special cases of the Extended Model.

In order to provide the assessment and management metric structure for corrective actions during and after a test, we define two types of B modes. Type BC failure modes are corrected during test. Type BD failure modes are delayed to the end of the test. Type A failure modes, as before, are those failure modes that will not receive a corrective action. These define the management strategy and can be changed. The Crow (AMSAA) model does not utilize the failure mode designation. This BC and BD failure mode designation is an important aspect of the Extended Model.

The test-fix-find-test concept is illustrated in Table 6. This is the same failure data as in Table 1, but Table 6 denotes those failure modes that received a corrective action during the test (BC modes) and also those failure modes that will receive a corrective action at the end of the test (BD modes). Note that in the failure mode designation BC modes are entirely different than BD modes. For example mode BC1 is an entirely different failure more from mode BD1 although both have a sub designation “1.” The test-fix-find-test strategy in Table 6 is to fix more failure modes than with the test-fix-test management strategy. During test the Type A and Type BD failure modes do not contribute to reliability growth. The corrective actions for the BC failure modes affect the increase in the system reliability during the test and this is the same for both Table 1 and Table 6 management strategies. After the incorporation of corrective actions for the Type BD failure modes, the reliability increases. Estimating this increased reliability with test-fix-find-test data is the objective of this paper.

For the Extended Model we assume that the achieved MTBF, before delayed fixes, based on Table 6 data should be exactly the same as the achieved MTBF for the Table 1 data. If K is the total number of distinct BD modes then, in Ref.5, the intensity to be estimated is

$$\lambda_P = \lambda_S - \lambda_B + \sum_{i=1}^{K}(1-d_i)\lambda_i + dh(T). \tag{30}$$

To allow for BC failure modes in the Extended Model we replace λ_S by λ_{CA} in (30). Also, let λ_{BD} be the constant failure intensity for the Type BD failure modes, and let $h(t)$ be the first occurrence function for the Type BD failure modes (see (10).

The Extended Model projected failure intensity is

$$\lambda_{EM} = \lambda_{CA} - \lambda_{BD} + \sum_{i=1}^{K}(1-d_i)\lambda_i + dh(T). \tag{31}$$

The Extended Model projected MTBF is $M_{EM} = 1/\lambda_{EM}$ This is the MTBF after the incorporation of the delayed BD failure modes that we wish to estimate.

Under the Extended Model the achieved failure intensity, before the incorporation of the delayed BD failure modes, is the first term λ_{CA}.The achieved MTBF at time T before the

BD failure modes is $M_{CA} = [\lambda_{CA}]^{-1}$. That is, the achieved MTBF before delayed fixes for the data in Table 6 is exactly the same as the achieved MTBF for the data in Table 1.

5.1. Estimation for the Extended Reliability Growth Model

The estimate of the projected failure intensity for the Extended Model is

$$\hat{\lambda}_{EM} = \hat{\lambda}_{CA} - \hat{\lambda}_{BD} + \sum_{j=1}^{M} (1-d_j)\frac{N_j}{T} + \bar{d}\,\hat{h}(T) \qquad (32)$$

where the first term is the Crow (AMSAA) model estimate (see Eq. (10)) applied to all A, BC, and BD data, as in Example 1, and the remaining terms are calculated as in Example 3, using only BD data as indicated in Table 4 and Table 5.

If it is assumed that no corrective actions are incorporated into the system during the test (no BC failure modes), then this is equivalent to assuming that $\beta = 1$ for $\lambda_{CA,}$, (see Eq. (10)), and λ_{CA} is estimated as in Example 1. In general, the assumption of a constant failure intensity ($\beta = 1$) can be assessed by a statistical test from the data

Example 4. Test-Fix-Find-Test

In Table 6 there are 56 total failures, T=400, and the failure times are the same as in Example 1. Table 6 will be used for several examples. For the current example it is assume that all the failure times X_J are known. There are BC failure modes but assume in this example that only the BD failure modes are designated. Assume the remaining Type A and Type BC failure modes are not designated as such.

The first term, $\hat{\lambda}_{CA}$, in $\hat{\lambda}_{EM}$ (Eq. (31) uses all failure time data in Table 4, as in Example 1. This gives (see Eq. (5))

$$\hat{\lambda}_{CA} = 0.1274\,. \qquad (33)$$

For the remaining terms in Eq. (31) the BD data in Table 4, and the EFs given in Table 3, are used. This Type BD data is the same Type B data in Example 2 so the calculations in Eq. (17) are the same. That is,

M=16, T= 400, $\lambda_{BD} = 0.08, \bar{d} = 0.72$. Also,

$$\sum_{j=1}^{M} (1-d_J)\frac{N_J}{T} = 0.0196 \;, \qquad (34)$$

and

$\hat{h}(T)$ has parameters $\hat{\lambda} = 0.1820$, $\hat{\beta} = 0.7472$.

This gives $\bar{d}\,h(T) = 0.0215.$ Therefore,

$$\hat{\lambda}_{EM} = 0.1274 - 0.08 + 0.0196 + 0.0215 \qquad (35)$$

or

$$\hat{\lambda}_{EM} = 0.0885. \qquad (36)$$

The Extended Model projected MTBF is $\hat{M}_{EM} = 11.29$. The achieved MTBF before the 16 delayed fixes is estimated by $\hat{M}_{CA} = 7.84$. We therefore have based on the Extended Model estimates that the MTBF grew to 7.84 as a result of corrective actions for BC failure modes during the test, and then jumped to 11.29 as a result of the delayed corrected actions after the test for the BD failure modes.

J	X_J	Mode	J	X_J	Mode
1	0.7	BC1	29	192.7	BD11
2	3.7	BC1	30	213	A
3	13.2	BC1	31	244.8	A
4	15	BD1	32	249	BD12
5	17.6	BC2	33	250.8	A
6	25.3	BD2	34	260.1	BD1
7	47.5	BD3	35	263.5	BD8
8	54	BD4	36	273.1	A
9	54.5	BC3	37	274.7	BD6
10	56.4	BD5	38	282.8	BC11
11	63.6	A	39	285	BD13
12	72.2	BD5	40	304	BD9
13	99.2	BC4	41	315.4	BD4
14	99.6	BD6	42	317.1	A
15	100.3	BD7	43	320.6	A
16	102.5	A	44	324.5	BD12
17	112	BD8	45	324.9	BD10
18	112.2	BC5	46	342	BD5
19	120.9	BD2	47	350.2	BD3
20	121.9	BC6	48	355.2	BC12
21	125.5	BD9	49	364.6	BD10
22	133.4	BD10	50	364.9	A
23	151	BC7	51	366.3	BD2
24	163	BC8	52	373	BD8
25	164.7	BD9	53	379.4	BD14
26	174.5	BC9	54	389	BD15
27	177.4	BD10	55	394.9	A
28	191.6	BC10	56	395.2	BD16

Table 6. Test-Fix-Find-Test Failure Times and Failure Mode Designations

6.0 *THE EXTENDED CONTINUOUS EVALUATION MODEL for PROJECTIONS ON BC and BD MODES*

The previous models given a projection based on the incorporation of corrective actions for BD models only. That is, a projection on failure modes corrected at the end of the test. We may also be interested in a projection for failure modes that may be corrected before the end of the test. The Extended Continuous Evaluation reliability growth model allows for projections on both BD modes and BC modes.

Suppose the reliability growth testing is specified for 3500 hours and at time T = 400 hours into the testing, the testing is stopped in order to update the system configuration with corrective actions. That is, the end of a test phase. Between time 0 and time T = 400 corrective actions were incorporated for some of the failure modes and additional corrective actions are planned to be incorporated at time T = 400. With the Extended Continuous Evaluation model assessments can be made throughout the testing. After the testing is resumed at time 400, additional assessments can be made and at time T = 3500 all remaining BD modes must all be corrected.

At an assessment point, for example T = 400, the corrective action status of all failure modes seen to-date is updated, as in Table 1. A failure mode corrected at the time of failure is denoted by a BC mode. Any failure mode that is not planned to be corrected during the 3500 hour test is denoted by an A mode. All failure modes that are planned to be corrected after the time of first occurrence are denoted as a Type BD mode. Among the BD modes those that have been corrected by the stop of the testing at time T = 400 are further denoted as a BDC mode. Those BD modes that have not been corrected by the stop of the testing at time T = 400 are denoted as a BDD mode. Of these BDD modes we can now chose to incorporate corrective actions for some before testing resumes, and defer corrective actions for the others to a later time between T = 400 and up to and including T = 3500. At time T = 400 we wish to get an updated assessment of our progress and the impact of the proposed corrective actions.

In Table 7, there are a total of 17 distinct Type BD modes. Of these 17 failure modes 5 have been corrected (Type BDC modes). The BDC modes are 1, 9, 10, 15 and 16. There are 12 Type BD modes not yet fixed (Type BDD modes). These are modes 2, 3, 4, 5, 6, 7, 8, 11, 12, 13, 14, 17. After the testing has stopped at time T = 400 we will incorporate corrective actions for BDD modes 2, 4, 6, 11, 14, 17. After these corrective actions are incorporated and testing resumed (T > 400) each of these BDD modes will be

Time to Failure	Failure Mode Status	Failure Mode Name	Time to Failure	Failure Mode Status	Failure Mode Name
0.7	BDC	1	192.7	BDD	12
15	BDD	2	213	A	35
17.6	BC	23	244.8	A	36
25.3	BDD	3	249	BDD	13
47.5	BDD	4	250.8	A	35
54	BDD	5	260.1	BDD	2
54.5	BC	24	273.1	A	35
56.4	BDD	6	274.7	BDD	6
63.6	A	34	282.8	BC	32
72.2	BDD	5	285	BDD	14
99.2	BC	25	315.4	BDD	4
99.6	BDD	7	317.1	A	34
100.3	BDD	8	320.6	A	36
102.5	A	34	324.5	BDD	12
112	BDC	9	324.9	BDC	10
112.2	BC	26	342	BDD	5
120.9	BDD	2	350.2	BDD	3
121.9	BC	27	355.2	BC	33
125.5	BDC	10	364.6	BDC	10
133.4	BDD	11	364.9	A	35
151	BC	28	366.3	BDD	2
163	BC	29	379.4	BDC	15
174.5	BC	30	389	BDC	16
177.4	BDC	10	394.9	A	36
191.6	BC	31	395.2	BDD	17

Table 7. Failure Times in First 400 Hours of Test, Failure Mode Status, and Failure Mode Name.

denoted as a BDC mode, that is, delayed and now corrected. Type BD modes 3, 5, 7, 8, 12, 13 are still denoted as BDD modes until corrected.

In Table 8 we assigned the Nominal Fix Effectiveness Factors for all the 12 Type BDD modes. A Nominal Fix Effectiveness Factor is the expected fraction decrease in the failure mode failure rate after a corrective action is implemented. There is also an assigned Actual Fix Effectiveness Factor which is equal to the Nominal Fix Effectiveness Factor if the corrective action is implemented at time T = 400 and is equal to zero if the corrective action is not implemented at time T = 400. Let d_N and d_A denote the average of the nominal and actual effectiveness factors. In the example

$$d_N = 0.699 \tag{37}$$

and

$$d_A = 0.346 . \tag{38}$$

Time to First Failure	Failure Mode Status	Failure Mode Name	Nominal EF	Actual EF
15	BDD	2	0.67	0.67
25.3	BDD	3	0.72	0
47.5	BDD	4	0.77	0.77
54	BDD	5	0.77	0
56.4	BDD	6	0.87	0.87
99.6	BDD	7	0.92	0
100.3	BDD	8	0.5	0
133.4	BDD	11	0.74	0.74
192.7	BDD	12	0.70	0
249	BDD	13	0.63	0
285	BDD	14	0.64	0.64
395.2	BDD	17	0.46	0.46

Table 8. Failure Times and Nominal and Actual Effectiveness Factors for the Distinct BDD Failure Modes

Each time we make an assessment with the Extended Continuous Evaluation model we can calculate each of these metrics:

a) Current Demonstrated MTBF
b) Nominal Growth Potential
c) Nominal Average EF
d) Nominal Projection if BDD modes are corrected with Nominal EFs
e) Actual Growth Potential
f) Actual Average EF
g) Actual Projection if BDD modes are corrected with Actual EFs
h) Rate of discovery.

The current demonstrated MTBF addresses the question: What is the reliability that is currently being demonstrated at time T = 400? This is given by the Crow (AMSSA) model. (See Section 2.) In Table 6 there are N = 50 total failures. Let X_i denote the i-th failure time in Table 1, i=1,…,50. Then, (using the unbiased estimate of beta) the Crow (AMSAA) model failure intensity at T = 400 hours is

$$\hat{r}_{CA} = \hat{\lambda}\hat{\beta}T^{\hat{\beta}-1} \quad (39)$$

where

$$\hat{\beta} = \frac{N-1}{\sum_{J=1}^{N}\ln(\frac{T}{X_i})}, \quad (40)$$

and

$$\hat{\lambda} = \frac{N}{T^{\hat{\beta}}}. \quad (41)$$

Example 5. Extended Continuous Evaluation Model.

We illustrate the Extended Continuous Evaluation model using the data in Tables 7 and 8. In this example,

$$\hat{\beta} = .9669, \quad (42)$$

$$\hat{\lambda} = .1524 \quad (43)$$

and the demonstrated failure intensity is

$$\hat{r}_{CA} = 0.1209. \quad (41)$$

The demonstrated, or achieved MTBF, is

$$MTBF_{CA} = \frac{1}{\hat{r}_{CA}} = 8.27. \quad (45)$$

If we put corrective actions into the system for some of the BDD modes what is our estimate (projection) of the new reliability? The Nominal Projection is the estimate if we fix all of the BDD modes at time 400, and the Actual projection is the estimate if we only fix those that have designated.

We first calculate the Nominal Growth Potential Factor (see Ref. 6)

$$\lambda_{NGPFactor} = \sum_{i=1}^{K}(1-d_i^{Nom})\frac{N_i}{T} \quad (46)$$

where K is the total number of distinct BDD modes at time T = 400, d_i^{Nom} is the assigned Nominal EF for the i-th BDD mode at time T =400, and N_i is the total number of failures over (0, 400) for distinct BDD mode i. In the example K = 12, and the EFs are given in Table 8.

In the example,

$$\lambda_{NGPFactor} = \sum_{i=1}^{K}(1-d_i^{Nom})\frac{N_i}{T} = 0.015000. \quad (47)$$

Next, we calculate the estimate of p(400)**.** This estimate is the total number of distinct BDD modes at time T = 400 divided by the total number of distinct BDC modes plus the total number of distinct BDD modes. In this example p(400) = (12/17) = 0.7059

The BDD mode failure intensity at time **T**= 400 is λ_{BDD} = total number of BDD failures (classification at the end of test at T = 400) divided by time 400. In the example

$$\lambda_{BDD} = 21/400 \quad (48)$$

or

$$\lambda_{BDD} = 0.0525. \quad (49)$$

The discovery function $h(t) = \lambda\beta t^{\beta-1}$ is calculated using all the first occurrences of the 17 BD modes. This is the rate in which new, distinct, Type BD modes are being discovered during the test. The M = 17 first failure times, Z_i, corresponding to discovering a new, distinct, Type BD mode (both BDC and BDD modes) are .7, 15, 25.3, 47.5, 54, 56.4, 99.6, 100.3, 112.0, 125.5, 133.4, 192.7, 249.0, 285.0, 379.4, 389.0, 395.2.

The unbiased estimate of beta for the h(t) function is:

$$\beta^* = \frac{M-1}{\sum_{i=1}^{M}\ln\left(\frac{T}{Z_i}\right)} = \frac{16}{\sum_{i=1}^{17}\ln\left(\frac{400}{Z_i}\right)} = 0.6055. \quad (50)$$

Beta must be less than one for reliability growth.

The h(t) function at time T = 400 is

$$h(400) = \beta^*\frac{M}{T} = 0.0257. \quad (51)$$

The classification of the A, BC and BD modes and the effectiveness factors define the management strategy. The growth potential is the maximum reliability that can be attained with the current management strategy. The nominal growth potential is the growth potential if all BD modes in the system were seen and corrected with the nominal effectiveness factors. The actual growth potential is the growth potential if all BD modes in the system were seen and corrected with the actual effectiveness factor strategy that has been demonstrated up to time T.

The Nominal Growth Potential failure intensity is

$$\lambda_{NGP} = \lambda_D - \lambda_{BDD} + \lambda_{NGPFactor} + d_N \cdot p \cdot h(T) - d_N \cdot h(T) \quad (52)$$

For this example,

$$\lambda_{NGP} = 0.0781 \quad (53)$$

The Nominal Growth Potential MTBF is

$$M_{NGP} = \frac{1}{\lambda_{NGP}} \quad (54)$$

For this example,

$$M_{NGP} = 12.8 \quad (55)$$

The Nominal Growth Potential MTBF is an estimate of the maximum reliability that is attainable for the system with the current design and management strategy. The overall system MTBF requirement or goal must be below the Nominal Growth Potential MTBF.

The Nominal Projection estimates the failure intensity and MTBF if all seen BDD modes are corrected at time T. The Nominal Projected Failure Intensity at time T is

$$\lambda_{NP} = \lambda_{NGP} + d_N \cdot h(T) \quad (56)$$

In the example

$$\lambda_{NP} = 0.0961. \quad (57)$$

The Nominal Projected MTBF at time T is

$$MTBF_{NP} = \frac{1}{\lambda_{NP}} \quad (58)$$

In the example the Nominal Projected MTBF is

$$MTBF_{NP} = 10.4 \quad (59)$$

If all 12 BDD modes 2, 3, 4, 5, 6, 7, 8, 11, 12, 13, 14, 17 were fixed at time T = 400 then the estimate of the new MTBF is 10.4, given by the Nominal Projected MTBF. However, we are only going to fix BDD modes 2, 4, 6, 11, 14 and 17 at time T = 400. The Actual Projected MTBF is the estimate of the new MTBF after we fix only modes 2, 4, 6, 11, 14 and 17 at time T = 400.

The Actual Growth Potential Factor is

$$\lambda_{AGPFactor} = \sum_{i=1}^{K} (1 - d_i^{Act}) \frac{N_i}{T} \quad (60)$$

where K is the total number of distinct BDD modes at time T, d_i^{Act} is the assigned actual EF for the i-th BDD mode at time T and N_i is the total number of failures over $(0, T_j)$ for distinct BDD mode i. In the example K = 12, and the actual EFs are in Table 2. In the example,

$$\lambda_{AGPFactor} = \sum_{i=1}^{K} (1 - d_i^{Act}) \frac{N_i}{T} = 0.03300. \quad (61)$$

The Actual Growth Potential Failure Intensity is

$$\lambda_{AGP} = \lambda_D - \lambda_{BDD} + \lambda_{AGPFactor} + d_A \cdot p \cdot h(T) - d_A \cdot h(T) \quad (62)$$

For this example,

$$\lambda_{AGP} = 0.0987. \quad (63)$$

The Actual Growth Potential MTBF is

$$M_{AGP} = \frac{1}{\lambda_{AGP}}. \quad (64)$$

For this example,

$$M_{AGP} = 10.13. \quad (65)$$

The Actual Growth Potential MTBF is an estimate of the maximum reliability attainable for the system with the current design and the management strategy that is demonstrated by the actual effectiveness factors.

The Actual Projected Failure Intensity at time T is

$$\lambda_{AP} = \lambda_{AGP} + d_A \cdot h(T) \quad (66)$$

In the example,

$$\lambda_{AP} = 0.1076. \quad (67)$$

The Actual projected MTBF at time T is

$$MTBF_{AP} = \frac{1}{\lambda_{AP}} \quad (68)$$

In the example, the Actual projected MTBF

$$MTBF_{AP} = 9.29 \quad (69)$$

Graphically, these results are given in Figure 2.

These calculations can be updated continuously throughout the entire reliability growth test and across the test phases. Also, the model can be implemented using grouped data over the test phases. At the completion of the reliability growth test all remaining BDD modes would be expected to be corrected.

7. *RELIABILITY GROWTH MODEL AND ESTIMATION PROCEDURES FOR DISCRETE DATA*

Discrete reliability growth models apply to systems, such as missiles, which are used one time. When the systems are operated the resulting outcome for each trial is either success or failure. These systems are often called “one shot” systems. The model considered in this section is the discrete version of the Crow (AMSAA) model.

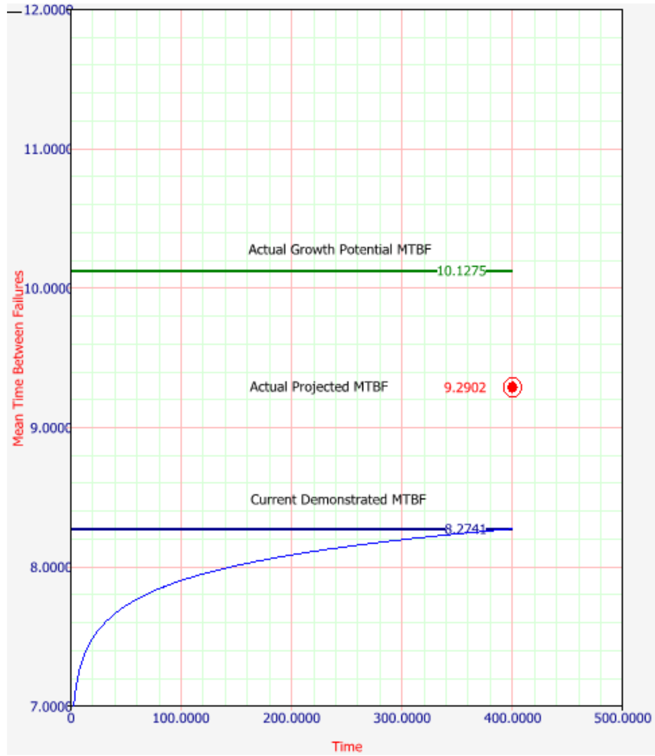

Figure 2. The Demonstrated MTBF , Actual Projected MTBF and Actual Growth Potential MTBF at $T = 400$

7.1 Discrete Reliability Growth Models

The Crow (AMSAA) Discrete Model (see Ref. 16) applies to one shot systems and assumes that reliability growth takes place on a configuration by configuration basis. That is, for configuration 1 of the system under development N_1 copies are made and tested. The number of failures in the N_1 trails is denoted by M_1 . Based on these failures corrective actions are introduced into the system and the updated design is configuration 2. For configuration 2 N_2 copies are made and tested. The number of failures observed for configuration 2 is M_2. This process is continued for K configurations and based on the data it is desired to estimate the reliability of the K-th configuration. The reliability of configuration K represents the current reliability of the system.

Let T_i be the cumulative number of trials through the i-th configuration, i = 1,…,K. That is $T_1 = N_1$, $T_2 = N_1 + N2$, etc.. For the Discrete Model the failure probability for the i-th configuration is given by

$$f_i = \frac{\lambda T_i^{\beta} - \lambda T_{i-1}^{\beta}}{N_i} \quad (70)$$

i = 1,…,K. The reliability for the i-th configuration is given by

$$R_i = 1 - f_i, \quad i = 1,\dots,K.$$

Based on the success-failure data for the K configurations the estimates of the parameters of the model are given by $\hat{\lambda}$ and $\hat{\beta}$ that satisfy the following equations:

$$\sum_{i=1}^{K} H_i \times S_i = 0 \quad (71)$$

$$\sum_{i=1}^{K} U_i \times S_i = 0 \quad (72)$$

where,

$$H_i = \left[T_i^{\hat{\beta}} \ln T_i - T_{i-1}^{\hat{\beta}} \ln T_{i-1}\right] \quad (73)$$

$$U_i = \left[T_i^{\hat{\beta}} - T_{i-1}^{\hat{\beta}}\right] \quad (74)$$

$$S_i = \left[\frac{M_i}{\hat{\lambda}T_i^{\hat{\beta}} - \hat{\lambda}T_{i-1}^{\hat{\beta}}} - \frac{N_i - M_i}{N_i - \hat{\lambda}T_i^{\hat{\beta}} + \hat{\lambda}T_{i-1}^{\hat{\beta}}}\right] \quad (75)$$

Example 6. Discrete Data

Suppose a missile system undergoes reliability growth development testing for a total of 68 trials. Delayed corrected actions were incorporated after the 14th, 33rd and 48th trials. From trial 49 to trial 68 the configuration is not changed. For this system K = 4, T_1 = 14, T_2 = 33, T_3 = 48, and T_4 = 68. That is,

Configuration 1 N_1 = 14 Trials.
Configuration 2 N_2 = 19 Trials.
Configuration 3 N_3 = 15 Trials.
Configuration 4 N_4 = 20 Trials.

The following failures were recorded for the four configurations:

Configuration 1 M_1 = 5 failures.
Configuration 2 M_2 = 3 failures.
Configuration 3 M_3 = 4 failures.
Configuration 4 M_4 = 4 failures.

Solving for the estimates of λ and β gives

$\lambda = 0.595$, and $\beta = 0.780$. (76)

These estimates give

$f_1 = 0.333$, $f_2 = 0.234$, $f_3 = 0.206$, $f_4 = 0.190$ (74)

and

$R_1 = 0.667$, $R_2 = 0.766$, $R_3 = 0.794$, $R_4 = 0.810$. (77)

That is, the reliability for the current configuration four is 0.810 from an initial reliability of 0.667 for configuration one.

7.2 Extended and Extended Continuous Evaluation Models for Discrete Data

The Extended model and the Extended Continuous Evaluation model have discrete versions that are applicable to one-shot data. These models give all of the estimates and projections that are available when applied to continuous data.

8. *Historical Data and Experiences for Key Reliability Growth Parameters*

In this section we will give historical information and experiences regarding (1) Duane/Crow (AMSAA) Growth Rates, (2) Fix Effectiveness Factors and (3) Discovery Rates.

In 1990 P. Ellner and P. Trapnell, Ref. 9 published average growth rates for a number of Army systems that had been subjected to reliability growth testing. The data in Table 9 was published in the 1990 Army study and the growth rates were calculated using the Crow (AMSAA) model applied to the data, with the growth rate alpha equal to $1-\beta$.

System Type	Growth Rate alpha
Helicopter	0.40
Navigation System	0.26
Navigation System	0.53
Navigation System	0.24
Ground Radio	0.40
Airborne Radio	0.32
Missile Electronic Sys	0.32
Missile 1	0.46
Missile 2	0.49
Missile 3	0.27
Missile 4	0.64
Missile 5 Test 1	0.61
Missile 5 Test 2	0.51
Missile 5 Test 3	0.32
Missile 6	0.60
Average	0.42

Table 9. Historical Data on Growth Rates

In Crow Ref, 7, actual fix effectiveness factors for a complex commercial electronic system being developed at Bell Laboratories were given. During the development program eleven fixes were incorporated into the system resulting in the system failure intensity being reduced to 1/5 of its original level. This number was calculated from extensive testing before and after the corrective actions. Figure 3 show the failure intensity before and after the eleven corrective actions. For this system all failure modes were addressed by corrective actions.

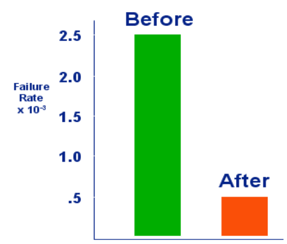

Figure 3. System Failure Rate Before and AfterCorrective Actions

Because of the extensive testing the actual FEFs were estimated for each of the eleven problem failure modes receiving corrective actions. These are given in Figure 4. The individual effectiveness factors ranged between a low of .13 to a high near 1.0, with an average of .80.

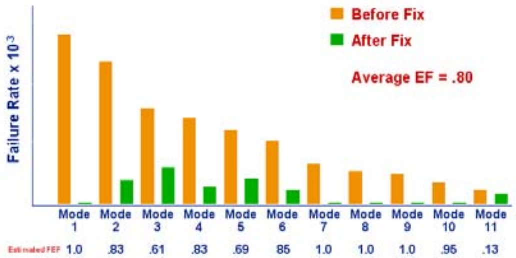

Figure 4. Failure Mode Failure Rates Before and AfterCorrective Action.

In 1990 P. Ellner and P. Trapnell (Ref. (9), also published average fix effectiveness factors (see also Ref. 8) for a number of Army systems that had been subjected to reliability growth testing. These are given in Table 10.

Heavy Ground Vehicles	FEF		Light Ground Vehicles	FEF
System 1	0.79		System 4	0.71
System 2	0.71		System 5	0.65
System 3	0.75		System 6	0.85
Avg. HGE	0.75		Avg. LGE	0.74
Missiles	FEF		Missiles	FEF
System 7	0.81		System 10	0.55
System 8	0.56		System 11	0.78
System 9	0.55			
			Avg. Missiles	0.65
Launchers	FEF		Misc.	FEF
System 12	0.60		System 14	0.75
System 13	0.66		System 15	0.61
			System 16	0.78
Avg. Launchers			Avg. Misc.	0.71

Table 10. Historical Data on Fix Effectiveness Factors

We next give information regarding the rate of discovery function. This information is not given in the 1990 Ellener and Trapnell report. For systems in the 1990 report the estimates regarding the discovery rate are derived, Table 12, from the data in the report using the Extended Reliability Growth Planning Model discussed next.

The discover function is given by equation (24). We write this discovery function in terms of parameters λ_D, β_D as

$$h(t) = \lambda_D \beta_D t^{\beta_D - 1}. \qquad (78)$$

The discovery betas in Table 11 were calculated directly from the data. These estimates of the discovery rate are directly estimated based on the times of first occurrence of problem failure modes and using the Crow (AMSAA) model. The average of these estimates is 0.70.

System Type	B_D
Military Complex Mechanical/Electronic Heavy	0.75
Military Complex Discrete (One Shot) System	0.61
Commercial Complex Mechanical/Electronic	0.66
Commercial Complex Mechanical/Electronic	0.77
Average	0.70

Table 13. Direct Estimates of the Discovery Rate Based on First Occurrence failure Mode Data and the Crow (AMSSA) Model.

The Growth Potential is the maximum reliability that can be attained from the system design and management strategy. (See Ref. 6.) In the Extended Planning model this is $\lambda_{GP} = \lambda_A + (1-d)\lambda_B$ for the growth potential failure intensity. The design margin is the ratio of the Growth Potential MTBF to the Target or Goal MTBF. If the design is 1.5, for example, the target MTBF is reached at 2/3 of the Growth Potential. In practice this is a common assumption because the rate of growth past 2/3 of the growth potential is generally very difficult. The management strategy is the fraction of the total initial failure rate that is in the Type B failure modes and addressed by corrective action if seen. This is $\lambda_B / (\lambda_A + \lambda_B)$. This is typically expected to be high, at 0.95 or greater.

For a helicopter in the 1990 report and based on other information the Extended Planning Model estimate the helicopter discovery beta to be $\beta_D = 0.726$.

For other systems in the 1990 report conservative estimates of the discovery rate were estimated based on certain reasonable assumptions. These results are given in Table 12.

System Type	Discovery Rate β_D
Navigation System	0.73
Ground Radio	0.70
Airborne Radio	0.69
Missile Electronic Sys	0.68
Missile 1	0.60
Missile 2	0.79
Missile 3	0.76
Missile 4	0.83
Missile 5	0.70
Average	0.72

Table 12. Derived Discovery Rates for Military Systems

The average of all of these estimates is about 0.72. Therefore, for planning purposes a beta for discovery in the 0.70 range would be reasonable based on this data.

9. *RELIABILITY GROWTH PLANNING AND MANAGEMENT*

As noted earlier, the 1982 Projection model (Ref 5) is a special case of the Extended Projection Model (Ref. 13). An Idealized Reliability Growth Planning Curve is the reliability at time t if all Type B modes that have been seen at time t are fixed. This is what the 1982 projection Model estimates. This application of the 1982 Projection model as an Idealized Growth Curve concept was proposed by Crow (Ref. 1984). Using an average Fix Effectiveness Factor d we write the 1982 projection model as

$$\lambda_{\text{Projection}} = \lambda_A + (1-d)\lambda_B + d\cdot\lambda_D\beta_D t^{\beta_D - 1}. \quad (79)$$

For projections the parameters are estimated and the effectiveness factor is an input based on historical experiences. As noted by Crow (1984) the projection model can be used as an Idealized Growth Curve. This is the Extended Reliability Growth Planning Model as defined in terms of failure intensity as

$$\lambda(t)_{\text{Extended-Planning}} = \lambda_A + (1-d)\lambda_B + d\cdot\lambda_D\beta_D t^{\beta_D - 1} \quad (80)$$

and in term of MTBF as

$$M(t)_{\text{Extended-Planning}} = \left[\lambda_A + (1-d)\lambda_B + d\cdot\lambda_D\beta_D t^{\beta_D - 1}\right]^{-1} \quad (81)$$

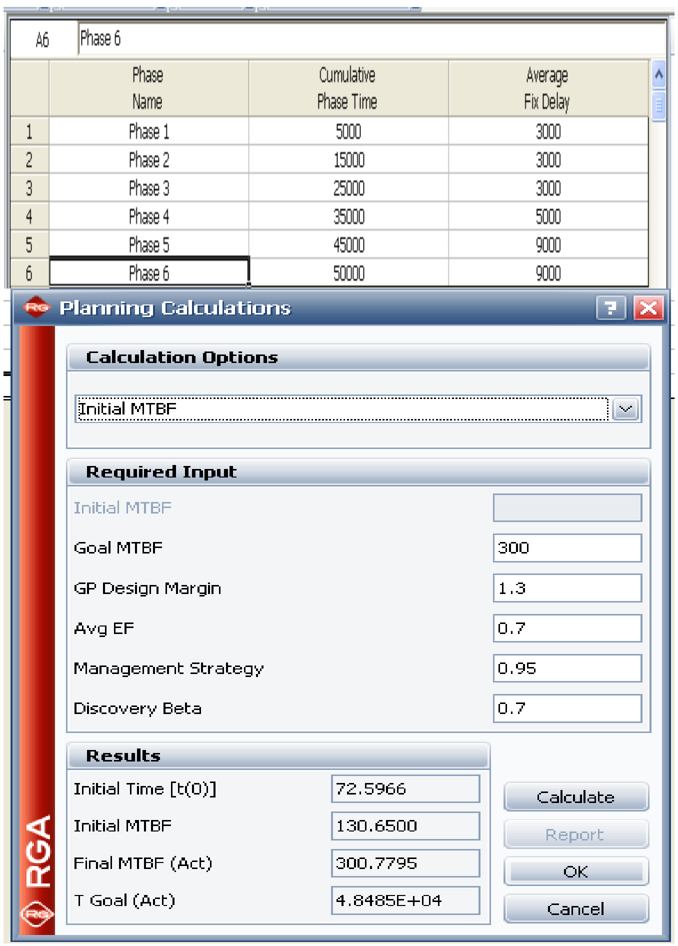

Figure 5. Planning Parameters for Reliability Growth Example.

For the Extended Reliability Growth Planning Model the key values are inputs for planning purposes. With the form of the discovery function in the Extended Reliability Growth Planning Model the discovery rate is characterized in terms of the discover rate parameter β_D . The Extended Reliability Growth Planning Model has the Learning Curve Property, equation (7), so that $\alpha = 1 - \beta_D$ may be viewed as the Growth Rate for the Extended Reliability Growth Planning Model.

Example 7.

Suppose for a vehicle we have a requirement of 300 miles MTBF. Engineering analyses indicate that a design with a growth potential for about 390 miles is cost effective and within the state of the art for this type of vehicle. Management has agreed to support a reliability growth test program up to a maximum of 50,000 miles. However, suppose these are expected average delays from the time a problem is first seen until the time a corrective action is incorporated, as indicated in Figure 5. With this design strategy we intend to mature the system design to about 75% of its growth potential that is the management strategy is about 1.3. With the inputs noted below, the Extended Planning model shows that the 300 mile requirement can be met at about 48,480 miles of reliability growth testing. This indicates that the 50,000 miles allocated should be sufficient and that the 300 mile requirement is attainable.

This gives the actual idealized and the planned cure in Figure 6.

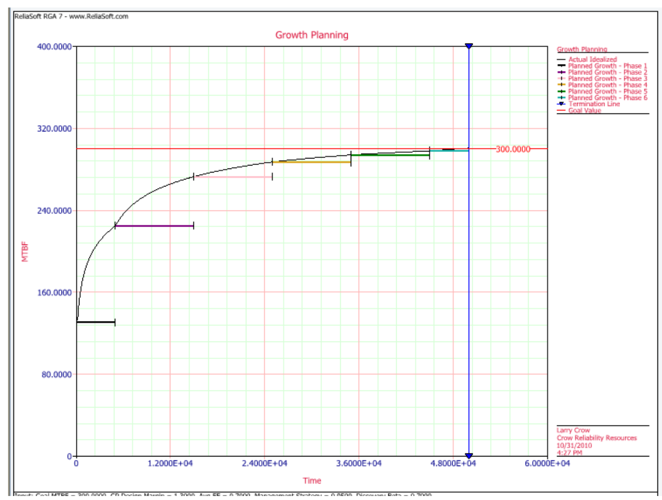

Figure 6. Actual Idealized Curve and Corresponding Planned Reliability Growth Curve.

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