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Modeling and Analysis for Degradation with an Initiation Time

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Key Words: degradation, initiation time, reliability importance.

SUMMARY & CONCLUSIONS

For failure modes that are caused by a degradation mechanism, almost all of the existing models assume that degradation starts once a product begins operation. However, under some situations, there is a degradation free period where degradation starts only after an initiation time. Both the initiation period and the degradation growth period affect the product reliability. The lengths of these two periods are usually governed by different failure mechanisms. In this paper, a two-step strategy for modeling degradation with an initiation period is proposed. Individual models are first built for the degradation free and the degradation growth periods. These two models are then integrated to obtain the final reliability model of the system. The relative importance values of the degradation free period and the degradation growth period with respect to product reliability are also studied. The importance values can help engineers allocate resources to improve the reliability of a product more effectively.

1 INTRODUCTION

Given that products are now designed to be highly reliable and that product development times need to be shorter, it is often not possible to test new designs to failure even under elevated stress conditions. However, in some cases, the amount of degradation can be determined from some measurable physical or performance characteristic. The product failure occurs when the amount of degradation reaches a critical level. In those cases, the degradation measurements can be used to extrapolate to assumed failure times, and then a time-to-failure distribution is estimated. Many different degradation models have been used for the extrapolation [1], and the common assumption is that the degradation mechanisms start once the product begins operation. However, this is not always the case. In many products there is a “degradation free” period where degradation starts only after an initiation time. For example, when testing automotive paint on metal, blisters will appear only after a certain initiation time and then begin to grow. Many other examples can be found such as failures caused by corrosion.

Clearly, this degradation initiation time is, by itself, a random variable. The ultimate failure of a product is a combined effect of the initiation time and the degradation growth. In those cases, in order to improve the reliability of a product, one can extend the degradation initiation time, reduce the degradation rate or do both. Therefore, it is equally important to study and quantify both the initiation time and the degradation path.

However, existing degradation modeling methods do not separate these two stages. Only a single distribution is used to fit the predicted failure times based on the degradation path [1], or a single distribution is used for modeling the degradation values and predicting the reliability [2]. In this paper, we propose a two-step strategy for modeling degradation with an initiation time. The degradation initiation time and the degradation path are modeled separately, and a method for integrating the two models and estimating the product reliability is proposed. From the proposed method, the relative importance of each stage can also be quantified, and thus provide direction for improving the product reliability by focusing on the degradation free stage, the degradation growth stage, or both.

2 MODELING STRATEGIES FOR DEGRADATION

First, the commonly used modeling strategy for degradation data is briefly explained, and then the proposed two-step strategy is introduced.

2.1 The Single Distribution Method

For degradation failure mechanisms like the wearout of tires, degradation starts once the tires are put to use. The degradation (or the thickness) of the tires is a function of time. Therefore, if the tires are inspected regularly, the amount of degradation can be recorded at each inspection time, and then a model, such as linear, power, or exponential, can be used to describe the degradation path. For example, the exponential model is:

$$y(t) = a \exp(bt) \quad (1)$$

where t is the time, $y(t)$ is the degradation at time t , a and b are model parameters.

Failure is defined when $y(t)$ reaches a pre-defined critical value y_c . For the i th sample under test, its failure time is:

$$t_i = 1/b_i \times \ln(y_c/a_i) \quad (2)$$

If there are n samples, n failure times will be obtained and used to fit a failure distribution. From this distribution, the reliability at a given time t can be calculated. For example, for the commonly used log-location-scale distributions, the reliability function is:

$$R(t) = 1 - \Phi\left(\frac{\ln(t) - \mu}{\sigma}\right) = \int_t^{\infty} \frac{1}{x\sigma} \phi\left(\frac{\ln(x) - \mu}{\sigma}\right) dx \quad (3)$$

where μ is the location parameter, σ is the scale parameter, Φ is the *cdf* (cumulative distribution function) and ϕ is the *pdf* (probability density function) for the standard log-location-scale distribution. If the life distribution is Weibull, then Φ is the standard SEV (smallest extreme value) distribution; if the life distribution is lognormal, then Φ is the *cdf* for the standard normal distribution.

Another single distribution method is used by Dr. Nelson. He assumed that the degradation readings follow a log-location-scale distribution, and the location parameter is a function of time [2]. For a given t , the distribution of the degradation readings can be obtained. From this distribution, one can calculate the probability that the degradation value will be less than (or beyond) a critical value. This probability is the predicted reliability of the product at time t .

All the single model methods do not separate the degradation initiation stage and the degradation growth stage, and thus do not allow us to study each individual stage. In order to improve product reliability, it is sometimes crucial to separate them in order to isolate and identify factors affecting each stage. Once these factors are identified, actions can be taken to increase the length of the degradation free stage and reduce the degradation growth rate.

2.2 The Two-Step Modeling Method

For cases where the degradation occurs in two stages, models like equation (1) cannot be used directly. In order to consider the degradation free stage, equation (1) can be modified as:

$$y_i(t) = a_i \exp[b_i(t - t_{0,i})], \quad t > t_{0,i} \quad (4)$$

where $t_{0,i}$ is the initiation time for the i th unit under test. This modification works well if $t_{0,i}$ can be observed and the measurement error of the degradation reading is not significant. However, in practice, it is almost impossible to observe $t_{0,i}$ unless the degradation is measured continuously. We have also found that sometimes the measurement error is relatively large compared to the degradation reading. Therefore, it is hard to identify a narrow range of values for the initiation time. A better method other than the modified equation (4) should be used.

Similar to predicting failure times based on the critical degradation value, we can also *predict* the initiation time. The

time corresponding to the degradation value of zero is the predicted initiation time. The following equation is one of these models:

$$y_i(t) = a_i + \exp(b_i t) \quad (5)$$

The predicted initiation time is:

$$t_{0,i} = 1/b_i \ln(-a_i) \quad (6)$$

Other models such as linear, power, and logarithm function also can be used. In this paper, we use Eq.(5) for illustration.

2.2.1 Data Selection for the Degradation Model

If the failure mode of interest is the crack on a surface, it is not difficult to figure out that the non-zero crack length data should be used to fit equation (5). However, in some cases, determining which data to use is not straightforward. For example, consider the case of a pump that is considered failed when the output air pressure is below a certain threshold. Assume that the normal air pressure reading is 20 psi. The seal of the pump will wear out during operation, and air will eventually start leaking out when the thickness of the seal is reduced to a certain level. The output air pressure will then start to drop. Once the pressure drops to 15 psi, failure occurs. However, the variation of the measured air pressure from one inspection time to another is relatively high, and the measurement tool does not provide accurate enough readings. When there are no leaks, the readings are around 20 psi. Once the leaking starts, a decreasing trend can be identified although some of the readings may still be around 20 psi shortly after the leaking starts. For scenarios like this, where the degradation initiation time cannot be easily identified, there are two methods that can be utilized to obtain degradation data that can then be used in modeling.

1).

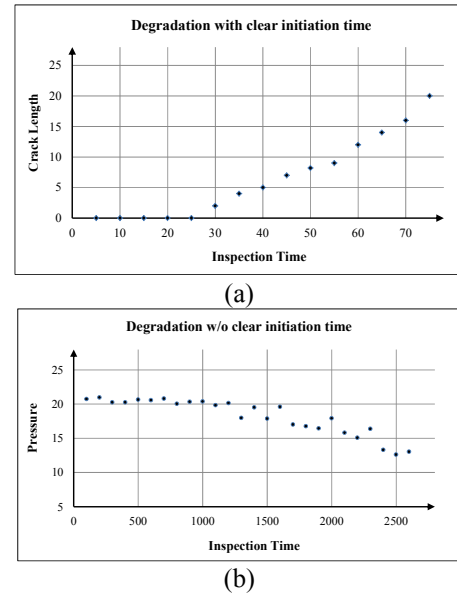


Figure 1. Degradation path with initiation time

1) If there are enough readings after the degradation starts, we can use only the observations that clearly belong to the degradation path, and discard the early readings that show

no observed degradation.

- 2) If there are not enough readings, or the variability between readings is high, we can use statistical methods such as a SPC (statistical process control) chart to identify the time when the departure from the average degradation reading is significant at a given significance level (which will constitute the initiation time) and use all the observations after this initiation time for the degradation model.

The preceding figures illustrate two common cases for degradation with initiation time. Figure 1 (a) shows a clear initiation time, while Figure 1(b) does not.

2.2.2 Reliability Modeling

From equation (5), two values can be obtained for each test unit: the predicted initiation time $t_{0,i}$, and the predicted failure time $t_{f,i}$. The degradation growth duration for the i th unit is:

$$t_{D,i} = t_{f,i} - t_{0,i} \quad (7)$$

Using $t_{0,i}$ from all the n test units, we can fit a distribution $f_1(t)$ to the initiation time data. Using $t_{D,i}$ for all the n test units, we can fit another distribution $f_2(t)$ to the degradation duration data. With these two distributions, the product reliability can be modeled by:

$$R(t) = R_1(t) + \int_0^t f_1(x) R_2(t-x) dx \quad (8)$$

where $R_1(t)$ is the probability that the degradation will not initiate by time t , and $R_2(t)$ is the probability that the degradation value will not beyond the critical value if degradation has initiated before time t . Since these two events are mutually exclusive, their sum is the total reliability of the product.

3 RELIABILITY IMPORTANCE AND IMPROVEMENT

Once the two stages in a degradation process are modeled separately, we can study the importance of each stage and identify which stage needs to be improved in order to raise the overall reliability.

3.1 Reliability Importance

The reliability importance index of a component within a system is defined as:

$$I_i(t) = \frac{\partial R(t)}{\partial R_i(t)} \quad (9)$$

Similarly, equation (9) can be used to define the relative importance of each stage of a degradation process. It can be proved that the relative importance of the degradation free stage is:

$$I_1(t) = F_2(t) = 1 - R_2(t) \quad (10)$$

The relative importance of the degradation growth stage is:

$$I_2(t) = F_1(t) = 1 - R_1(t) \quad (11)$$

The proof is given in Appendix A. These results make sense

intuitively since they show that one stage is relatively more important when the other one is not reliable.

3.2 Life-Variable Relation and Improving Reliability

For log-location-scale distributions, the location parameter μ represents the life of a product and is affected by design and stress variables. In order to improve the designed reliability, one needs to find out how different design variables affect product life. The relationship between life and these variables can be modeled as:

$$\mu = g(\underline{x}) \quad (12)$$

where $\underline{x} = (x_1, x_2, \dots, x_k)$ are the design variables, and $g(\underline{x})$ is a linear or non-linear function that represents the life-variable relationship. Once $g(\underline{x})$ is obtained either through DOE (design of experiments) or historical data, we can adjust the values of \underline{x} to improve the product's life [3, 4].

For the case of degradation with an initiation time where two stages are present, and in order to improve product reliability, we need to increase the length of both the degradation free (initiation) and degradation growth stages. In other words, we need to increase the location parameter μ for both distributions. However, if resources are limited and we can focus only on one stage (or one failure mechanism), the question then is, which μ of the two distributions should be increased first? Therefore, we need to study the importance of each μ . The importance of each μ is defined below.

$$l_i(t) = \frac{\partial R(t)}{\partial \mu_i} \quad (13)$$

To show μ in the reliability equation, equation (8) can be rewritten in terms of a standard log-location-scale distribution:

$$R(t) = 1 - \Phi(z_1(t)) + \int_0^t \frac{\phi(z_1(x))}{x\sigma_1} [1 - \Phi(z_2(t-x))] dx \quad (14)$$

where $z_i(t) = (\ln(t) - \mu_i) / \sigma_i$. Using equation (13), we get the importance index for μ_1

$$l_1(t) = \phi(z_1(t)) \frac{1}{\sigma_1} + \int_0^t \frac{z_1(x)}{x\sigma_1^2} \phi(z_1(x)) [1 - \Phi(z_2(t-x))] dx \quad (15)$$

and for μ_2

$$l_2(t) = \int_0^t \frac{z_1(x)}{x\sigma_1\sigma_2} \phi(z_1(x)) \phi(z_2(t-x)) dx \quad (16)$$

From equations (15) and (16), we can quantify the effect of each μ_i to the system reliability. As a result, we can identify which location parameter needs to be increased in order to improve the overall reliability.

4 ILLUSTRATIONS

This section provides an example that illustrates the proposed two-step modeling strategy and the reliability importance analysis for a degradation failure with initiation time.

4.1 Problem Statement

The surface quality of a product is affected by the manufacturing process and materials used. After a period of usage, cracks will appear on the surface. Failure occurs at the time when the crack length is larger than 120 mm. Twenty-five samples were tested and inspected every 5 days. The results are given in the next table.

Table 1. Degradation data for a product

Time	y_1	y_2	y_i	y_{25}
5	0	0		0
10	0	0		0
15	0	0		0
20	0	0		0
25	0	0		0
30	0	0		0
35	0	0		0.8
40	0	0		2
45	0	0		4
50	0	0		4.5
55	0	0	4.8	
60	0.6	7.9	...	7.9
65	7.6	25.4		16.4
70	17.6	42.3		25.4
75	25.3	62.8		36.5
80	40.2	101.3		52.9
85	51.8	152.8		64.9
90	76	226.3		93
95	102.6	337.3		131.4

For this example, we can clearly identify the time range when the degradation starts. Therefore, for each sample, we use only the data where a crack growth is presented in order to build a degradation model. For example, for y_1 , the data after an inspection time of 60 days is used because degradation starts somewhere between 55 and 60 days.

4.2 Modeling

The exponential function of equation (5) is used for each sample. The estimated model parameters are:

Table 2. Estimated parameters for degradation models

Sample	1	2	i	25
a	-18.6631	-38.1087	...	-10.0578
b	0.0505	0.0622		0.0517

The observed and predicted degradation values for each test sample are given in Figure 2t. Using the estimated model parameters, we can predict the initiation time and the failure time for each sample. From these two values, we can get the degradation growth duration using equation (7), as shown in Table 3.

The lognormal distribution is found to be a good fit for both the initiation time and the degradation growth duration. For the initiation time, the distribution parameters are $\mu_1 = 3.9032$, $\sigma_1 = 0.1366$. For the degradation growth duration, the distribution parameters are $\mu_2 = 3.5245$, $\sigma_2 = 0.3101$. The results are given in Figure 3.

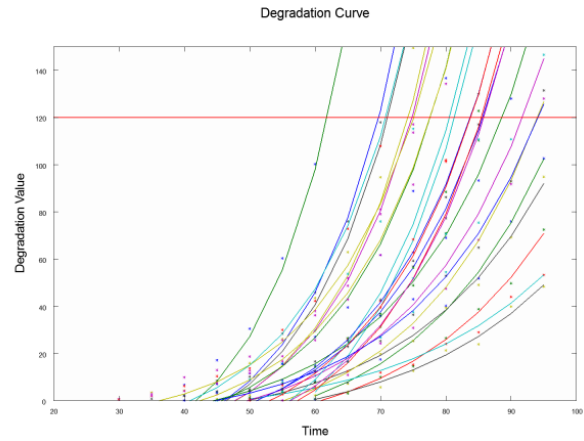


Figure 2. Observed and predicted degradation values

Table 3. Predicted initiation time and failure time

Sample	Initiation Time	Failure Time	Growth Duration
1	57.95	97.67	39.72
2	58.55	81.44	22.88
3	48.00	74.29	26.29
4	46.32	69.76	23.44
5	60.94	104.32	43.38
6	47.88	74.84	26.96
7	49.58	99.90	50.32
8	42.00	62.00	20.00
9	47.90	111.99	64.09
10	55.19	94.09	38.90
11	50.76	85.70	34.93
12	57.49	85.49	27.99
13	49.63	91.80	42.17
14	46.61	71.21	24.60
15	44.42	88.69	44.27
16	56.25	80.61	24.36
17	36.31	75.11	38.80
18	51.15	83.84	32.69
19	53.19	83.90	30.71
20	56.36	85.88	29.52
21	59.01	112.52	53.51
22	45.73	77.77	32.04
23	40.97	70.89	29.92
24	42.54	77.74	35.20
25	44.66	94.17	49.52

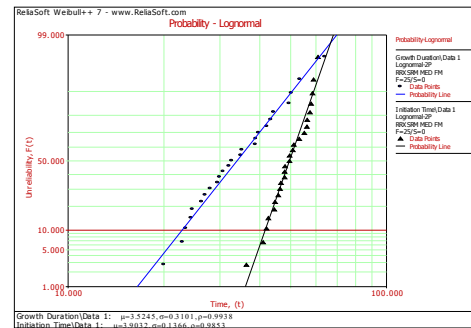


Figure 3. Probability plot of distributions for the degradation initiation time and degradation growth duration

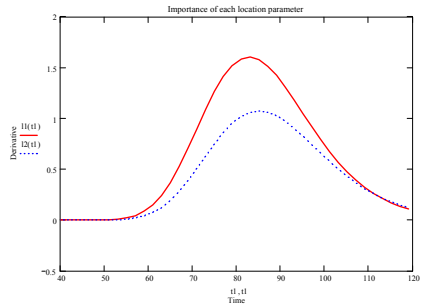
With these two distributions, the predicted reliability can be obtained for any given time t , using equation (14). For

example the reliability at 60 days is $R(60) = 0.9914$.

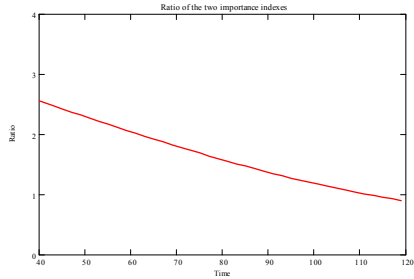
4.3 Reliability Importance Analysis

Equations (10) and (11) show that the unreliability of one stage is the reliability importance of the other stage. Therefore, the unreliability plot in Figure 3 shows that the initiation stage is more important to the system reliability than the degradation growth stage since the unreliability value for the growth stage is higher than the initiation stage.

The relative importance of the life parameter μ in each distribution can also be studied using equations (15) and (16). The values of the two derivatives $\partial R(t)/\partial \mu_1$ and $\partial R(t)/\partial \mu_2$ are plotted in Figure 4 (a). Their ratio is given in Figure 4 (b).



(a) Importance index for u_1 and u_2



(b) Ratio of $l_1(t)$ and $l_2(t)$

Figure 4. Importance of the location parameters for the two stages

From Figure 4, we can also see that increasing the location parameter of the distribution for the initiation time will have a bigger effect than increasing the location parameter for the growth duration. Therefore, as the first step, attention should be focused on the failure mechanism that increases the length of the degradation free period.

5 CONCLUSIONS

In this paper, we proposed a two-step modeling strategy for failures caused by degradation with an initiation time. This method is better than the single model method, which focuses only on the final failure times. The two-step method builds individual models for the degradation initiation time and the degradation growth period. These two models are integrated to obtain the final system reliability. From the system reliability, the reliability importance of each stage in the degradation process can be quantified. The importance indexes give us a better understanding of the effect of each stage on reliability,

and also provide direction for further improving product reliability, such as using design of experiments [3, 4].

APPENDIX

A. Proof for Equations (10) and (11)

The product reliability function of equation (8) can be rewritten as

$$R(t) = R_1(t) - \int_0^t R_1'(x)R_2(t-x)dx \quad (A.1)$$

Applying the partial integration for the second term, equation (A.1) becomes

$$R(t) = R_2(t) + \int_0^t R_1(x)f_2(t-x)dx \quad (A.2)$$

Define

$$R^\Delta(t) = R_2(t) + \int_0^t [R_1(x) + \Delta]f_2(t-x)dx \quad (A.3)$$

then

$$\frac{R^\Delta(t) - R(t)}{\Delta} = \int_0^t f_2(t-x)dx \quad (A.4)$$

Define $v = t - x$, equation (A.4) becomes

$$\frac{R^\Delta(t) - R(t)}{\Delta} = \frac{\partial R(t)}{\partial R_1(t)} = \int_0^t f_2(v)dv = F_2(t) \quad (A.5)$$

To prove equation (11), equation (8) can be rewritten as

$$R^\Delta(t) = R_1(t) + \int_0^t f_1(x)[R_2(t-x) + \Delta]dx \quad (A.6)$$

so we get equation (11) by

$$\frac{R^\Delta(t) - R(t)}{\Delta} = \frac{\partial R(t)}{\partial R_2(t)} = \int_0^t f_1(x)dx = F_1(x) \quad (A.7)$$

B. Proof for Equation (15) and (16)

Since

$$z_i(t) = \frac{\ln(t) - \mu_i}{\sigma_i}, \quad \frac{\partial R(t)}{\partial \mu_i} = \frac{\partial R(t)}{\partial z_i(t)} \frac{\partial z_i(t)}{\partial \mu_i} = -\frac{1}{\sigma_i} \frac{\partial R(t)}{\partial z_i(t)} \quad (B.1)$$

From equation (14), we know

$$\frac{\partial R(t)}{\partial \mu_1} = \frac{\phi(z_1(t))}{\sigma_1} + \int_0^t -\frac{\phi'(z_1(x))}{x\sigma_1^2} [1 - \Phi(z_2(t-x))] dx \quad (B.2)$$

Since

$$\phi'(z_1(x)) = -z_1(x)\phi(z_1(x))$$

we get:

$$\frac{\partial R(t)}{\partial \mu_1} = l_1(t) = \frac{\phi(z_1(t))}{\sigma_1} + \int_0^t \frac{z_1(x)}{x\sigma_1^2} \phi(z_1(x)) [1 - \Phi(z_2(t-x))] dx \quad (B.3)$$

Similarly, since

$$\phi(z_2(t)) = \frac{d\Phi(z_2(t))}{dz_2(t)}$$

From equation (14), we can have

$$\frac{\partial R(t)}{\partial \mu_2} = \int_0^t \frac{\phi(z_1(x))}{x\sigma_1} \left[\frac{\phi(z_2(t-x))}{\sigma_2} \right] dx \quad (B.4)$$

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