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A Method for Reliability Allocation with Confidence Level

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Key Words: reliability allocation, reliability diagram, reliability demonstration test

SUMMARY & CONCLUSIONS

One of the important reliability activities in Design for Reliability (DFR) is system reliability allocation at the early product design stage. Usually system reliability is given as product performance requirement under the normal use condition (e.g., the probability of failure for a 2-year operation should be less than 0.05 with a confidence level of 90%). Complex systems consist of many subsystems, which are developed concurrently and sometimes independently. It would be too late to validate the system reliability until the final system prototype is ready after months or years of development. From a project management point of view, the reliability for each subsystem or sub-function should be examined as early as possible. Therefore, allocating a reasonable reliability requirement to each subsystem and sub-function based on the system reliability requirement is very important. Many reliability allocation approaches can be used, such as the AGREE, weighted, equal reliability, and cost-based methods. Traditional system reliability allocation is conducted on the reliability requirement alone. None of the above existing methods considers allocating the reliability requirement together with a confidence level.

In this paper, we will propose a new approach for allocating system reliability together with confidence level to the subsystems. The proposed method can be used for complex systems with serial, parallel, and k-out-of-n configurations.

1 INTRODUCTION

The development cycle for many systems such as medical devices and military equipment usually is very long. The system reliability requirement should be allocated to each subsystem as early as possible in the product development stage. If each subsystem cannot meet its own reliability requirement, then the final system will not be able to meet the requirement. Due to this reason, many reliability allocation methods have been proposed in the past half century [1]. The simplest reliability allocation method is the so called equal reliability allocation. This method is used for a system with a serial reliability configuration. For example, assume a system has three independent subsystems A, B, and C. The system reliability is a function of the subsystem reliability, which is:

\[ R_S = R_A \times R_B \times R_C \]  

where \( R_S \) is the system reliability. If the required \( R_S \) is 0.90, the reliability of each subsystem will be:

\[ R_A = R_B = R_C = (R_S)^{1/3} = 0.9655 \]  

Once one of the subsystems is ready for testing, we can design a demonstration test to see if it meets the required reliability. For example, 30 samples for subsystem A are tested and 1 failure was obtained; does it meet the required reliability of 0.9655? The estimated reliability can be calculated as:

\[ \hat{R}_A = 29/30 = 0.9667 \]  

The above result shows that subsystem A meets the requirement. However, since \( R_A \) is estimated from the data, there is uncertainty associated with \( \hat{R}_A \). A large sample size will reduce the uncertainty of \( \hat{R}_A \). The variance of \( \hat{R}_A \) is:

\[ \text{Var}(\hat{R}_A) = \frac{\hat{R}_A(1-\hat{R}_A)}{n} \]  

Eqn. (3) is the so called Maximum Likelihood Estimate (MLE). Using Eqn. (3) and Eqn. (4), the confidence bounds of \( R_A \) at a given confidence level can be calculated [2].

Now if 20 samples are tested without any failure, does subsystem A meet the requirement? Clearly, the calculation in Eqn. (3) and (4) cannot be used when there is no failure. Otherwise, the estimated reliability will be 1 without any uncertainty since its variance is 0. Instead of using the MLE estimator, the following binomial equation is used for reliability estimation:

\[ 1-CL = \sum_{i=0}^{n} \binom{n}{i} (1-R)^i R^{n-i} \]  

where \( n \) is the sample size, \( r \) is the number of failures and \( CL \) is the confidence level. We know the sample size \( n \) is 20 and the number of failures is 0. So using these values, Eqn. (5) becomes:

\[ 1-CL = R^n \]

Without knowing a value for \( CL \), we cannot solve \( R \) from Eqn. (6). If we set \( CL = 0.5 \), then we have:

\[ 1-0.5 = R^{20} \]

\[ R = 0.0012 \]
1 - CL = R^n \Rightarrow 0.5 = R^{20} \Rightarrow R = 0.9659 \quad (7)

So at a confidence level of 0.5, the reliability meets the requirement of 0.9655. However, if we set \( CL = 0.95 \), then:

1 - CL = R^n \Rightarrow 0.05 = R^{20} \Rightarrow R = 0.8609 \quad (8)

The result shows that the reliability does not meet the requirement. Therefore, without giving a confidence level in the reliability requirement, we cannot answer the question: “Does A meet the reliability requirement?” The confidence level must be in the reliability requirement.

However, none of the existing reliability allocation methods can be applied to allocate the reliability with a confidence level. Very little research has been done on allocating system reliability requirement with a confidence level. In this paper, we will develop a new method to solve this important issue. The paper is arranged as follows: Section 2 will briefly review several commonly used reliability allocation methods first and then we will explain our new method. Section 3 will give two examples to illustrate how to apply the proposed method. Section 4 is the conclusion.

2 System Reliability Allocation with Confidence Level

From the above section, we can see that the system reliability is a function of its subsystem reliability. For a system with \( k \) subsystems/components, its reliability is:

\[
R_s = g(R_1, R_2, \ldots, R_k) \quad (9)
\]

where:
- \( R_j \) is the system reliability.
- \( R_i(i = 1, \ldots, k) \) is the reliability of subsystem \( i \).
- \( g \) is the function between \( R_i \) and \( R_s \).

If the system reliability requirement \( R_s^* \) is given, we need to find \( R_s \) to make sure:

\[
R_s^* \leq g(R_1, R_2, \ldots, R_k) \quad (10)
\]

Several methods for reliability allocation have been developed [1]. The simplest method is the equal allocation method we illustrated in Section 1. This method can only be applied when the system reliability configuration is in series. The system reliability is calculated by:

\[
R_s = \prod_{i=1}^{k} R_i \quad (10)
\]

The allocated reliability for each subsystem is:

\[
R_i = \left( R_s^* \right)^{1/k} \quad (11)
\]

Other methods such as the AGREE method, the ARINC method developed by the Advisory Group on Reliability of Electronic Equipment, and Penalty Function Based Allocation are also used. For detail, please refer to \([1, 3]\). None of the existing allocation methods consider confidence level. We will explain our allocation method for system reliability and confidence level next.

As discussed in Section 1, when reliability is estimated from data, the estimated reliability is a random variable which can be described by a distribution. For example, when the MLE estimate is used, \( \hat{R}_i \) is assumed to be normally distributed asymptotically, with mean of \( (1 - r_i)/n_i \) and variance as given in Eqn. (4), where \( r_i \) is the number of failures when \( n_i \) samples are tested [2]. When the binomial equation, Eqn. (5), is used to estimate the reliability, the estimated reliability \( \hat{R}_i \) follows a beta distribution \( \text{Beta}(R_i; n_i - r_i, r_i + 1) \) [4]. If the CL percentile of \( \hat{R}_i \) is greater than \( R_i^* \), a reliability requirement at confidence level \( CL \), then it means the reliability has met the requirement.

2.1 Reliability Requirement with a Confidence Level

The binomial formula of Eqn. (5) is commonly used in reliability demonstration test (RDT) design. It has four variables: \( R \), \( CL \), sample size \( n \), and number of failures \( r \). Since \( R \) and \( CL \) are given as part of the requirement, we need to find \( n \) and \( r \) in designing a demonstration test. For example, if the requirement is \( R = 90\% \) and \( CL = 90\% \), then there are an infinite number of combinations of \( n \) and \( r \) that can demonstrate the required reliability. Some of the combinations are given in Table 1.

<table>
<thead>
<tr>
<th>Number of Failures</th>
<th>Sample Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>22</td>
</tr>
<tr>
<td>1</td>
<td>38</td>
</tr>
<tr>
<td>2</td>
<td>52</td>
</tr>
<tr>
<td>3</td>
<td>65</td>
</tr>
<tr>
<td>4</td>
<td>78</td>
</tr>
<tr>
<td>5</td>
<td>91</td>
</tr>
</tbody>
</table>

Table 1: Test Plan for \( R=90\% \) and \( CL=90\% \)

From the above table, we can see that testing 22 samples without failure is equivalent to testing 38 samples with 1 failure in terms of demonstrating \( R = 90\% \) and \( CL = 90\% \). The zero failure test requires the smallest sample size, so it is the most popularly used test in RDT.

The system reliability is a function of the subsystem reliabilities, which are estimated from test data. Since the distribution of reliability is determined by the sample size and the number of failures, the reliability allocation problem is the same as determining the test plan for each subsystem. In other words, we need to calculate the system reliability at a given confidence level from the estimated subsystem reliability.

We will discuss how to calculate the system reliability distribution from subsystem reliability in the next section.

2.2 Mean and Variance for Component and System Reliability

The estimated reliability from Eqn. (5) for the \( i \)th component or subsystem is a beta distribution \( \text{Beta}(R_i; n_i - r_i, r_i + 1) \). The mean and variance of \( \hat{R}_i \) are:
The reliability of a serial system is given in Eqn. (10). Since \( R_i \) follows the beta distribution, \( R_s \) is the product of multiple independent beta random variables. The exact distribution for \( R_s \) can be found using the Mellin transform [5]. However, this calculation is very intensive. To simplify the calculation, Thompson and Haynes [6] suggested approximating the exact distribution with a beta distribution having the same first two moments. In fact, without getting the exact or approximated distribution for \( R_s \), its mean and variance can be calculated by [7]:

\[
E(\hat{R}_s) = \prod_{i=1}^{k} E(\hat{R}_i) = \prod_{i=1}^{k} \frac{a_i}{a_i + b_i} \tag{14}
\]

\[
Var(\hat{R}_s) = E(\hat{R}_s) \prod_{i=1}^{k} \frac{a_i + 1}{a_i + b_i + 1} - E^2(\hat{R}_s) \tag{15}
\]

where \( a_i = n_i - r_i, b_i = r_i + 1 \).

Eqn. (15) is based on the following conclusion. If \( X = \sum_{i=1}^{k} X_i \) and the mean and the variance for \( X_i \) are known, the variance for \( X \) can be estimated by [7]:

\[
Var(X) = \sum_{i=1}^{k} [E^2(X_i) + Var(X_i)] - \sum_{i=1}^{k} E^2(X_i) \tag{16}
\]

### 2) Parallel System Reliability

The reliability for a parallel system is calculated by:

\[
R_s = 1 - \prod_{i=1}^{k} (1 - R_i) = 1 - \prod_{i=1}^{k} F_i \tag{17}
\]

Since \( R_i \sim \text{Beta}(n_i - r_i, r_i + 1) \) and \( F_i = 1 - R_i \), \( F_i \) is also a beta distribution, \( F_i \sim \text{Beta}(F_i; r_i + 1, n_i - r_i) \). Formulas similar to Eqn. (14) and (15) can be used to get the mean and variance for \( R_s \) for a parallel system. They are:

\[
E(\hat{R}_s) = 1 - \prod_{i=1}^{k} E(\hat{F}_i) = 1 - \prod_{i=1}^{k} \frac{b_i + 1}{a_i + b_i + 1} \tag{18}
\]

\[
Var(\hat{R}_s) = \left[ 1 - E(\hat{R}_s) \right] \prod_{i=1}^{k} \frac{a_i + 1}{a_i + b_i + 1} - 1 + E(\hat{R}_s) \tag{19}
\]

where \( a_i = n_i - r_i + 1, b_i = n_i - r_i \).

Eqn. (19) is based on the following conclusion. If \( X = 1 - \prod_{i=1}^{k} (1 - X_i) \), the variance for \( X \) can be estimated by:

\[
Var(X) = \prod_{i=1}^{k} [E^2(1 - X_i) + Var(X_i)] - \prod_{i=1}^{k} E^2(1 - X_i) \tag{20}
\]

### 3) Complex System

A complex system is a system that can be composed of many series and parallel subsystems. By using the above equations iteratively for each subsystem, the system \( E(\hat{R}_s) \) and \( Var(\hat{R}_s) \) can be obtained. For detail, please see [4].

### 2.3 System Reliability Distribution Approximation

Once the \( E(\hat{R}_s) \) and \( Var(\hat{R}_s) \) are obtained, we will approximate the system reliability distribution by a beta distribution, \( \text{Beta}(R_s; a, b) \). For a random variable \( X \) following a beta distribution with parameters \( a \) and \( b \), its mean and variance are:

\[
E(X) = a/(a + b) \tag{21}
\]

\[
Var(X) = ab/[((a + b)^2(a + b + 1)] \tag{22}
\]

Setting Eqn. (21) and (22) to the estimated mean and variance obtained from the previous procedure for serial, parallel and complex systems, the parameters \( a \) and \( b \) can be estimated using:

\[
a = E(\hat{R}_s) \left[ \left( E(\hat{R}_s) - E^2(\hat{R}_s) \right)/Var(\hat{R}_s) - 1 \right] \tag{23}
\]

\[
b = \left( 1 - E(\hat{R}_s) \right) \left[ \left( E(\hat{R}_s) - E^2(\hat{R}_s) \right)/Var(\hat{R}_s) - 1 \right] \tag{24}
\]

Once \( a \) and \( b \) are found, the one-sided 100% CL lower bound for the system reliability \( R_{sL} \) can be calculated by:

\[
\int_0^{R_{sL}} \text{Beta}(R_s; a, b) dR_s = 1 - CL \tag{25}
\]

### 2.4 A General Method for System Reliability Allocation with Confidence Level

Given that the system reliability requirement is \( R_s^* \) at a confidence level of \( CL \), the calculated \( R_{sL} \) from Eqn. (25) should be greater than \( R_s^* \). Since the system reliability is a function of subsystem reliabilities, there are many different ways that reliability can be allocated without adding any constraints. In this paper, the following constraints are considered in the allocation:

- The confidence level for each subsystem is the same as the one for the system.
- All of the system and subsystem tests are zero-failure tests.

The above constraints fix \( CL_i = CL \) and \( r_i = 0 \). In the binomial equation:

\[
1 - CL_i = \sum_{i=0}^{n_i} \binom{n_i}{i} (1 - R_i)^i R_i^{n_i - i} \tag{26}
\]

only \( n_i \) and \( R_i \) are free to change. Allocating \( R_i \) is the same as determining \( n_i \), since they have the one-to-one relation when \( CL_i \) and \( r_i \) are fixed.

For different subsystems, the risk of a failure and the cost of testing a sample are different. Risk can be defined as the product of failure probability and severity of the failure consequence. For example, if a subsystem failure of a medical device will cause permanent injury to a patient, then its
severity is higher than a failure that only causes minor consequences (e.g. recoverable injury or inconvenience to patient). Subsystems with high risk should be allocated with higher reliability. The cost of conducting a test for one subsystem may be more expensive or take much longer time than other subsystems. If a very high reliability value is assigned to this subsystem, it may be impossible to demonstrate it, so the allocated reliability for each subsystem should be able to demonstrate. All such important constraints should be considered in reliability allocation. The cost or risk can be the penalty function in the reliability allocation. Therefore, the reliability allocation problem becomes:

\textbf{Minimize:} A penalty function

\textbf{Determine:} \( R_i \) at confidence level of CL

\textbf{st:}
- \( R_{si} \geq R_i^* \)
- Constraints on sample size of each subsystem, risk weighting factor, budget, etc.

Where: \( R_{sl} = \text{Inv Beta}(1-\text{CL}, a, b) \),

\[ a = f_a(R_1, R_2, \ldots, R_k) \quad \text{and} \quad b = f_b(R_1, R_2, \ldots, R_k) \quad \text{show that} \quad a \quad \text{and} \quad b \quad \text{for the beta distribution of the system reliability are functions of the subsystem reliabilities.} \]

2.5 \textit{A Simple Allocation Method for Serial Systems}

When the system reliability configuration is in series, the above allocation procedure can be simplified by adding some constraints. For example, we can require equal reliability at the same confidence level for each subsystem. The allocation problem becomes:

\textbf{Minimize:} \( R_{sl} \)

\textbf{Determine:} \( R_i \) at confidence level of CL

\textbf{st:} \( R_i = R_2 = \ldots = R_k \), \( R_{sl} \geq R^* \)

Where: \( R_{sl} = \text{Inv Beta}(1-\text{CL}, a, b) \),

\[ a = f_a(R_1, R_2, \ldots, R_k) \quad \text{and} \quad b = f_b(R_1, R_2, \ldots, R_k) \quad \text{show that} \quad a \quad \text{and} \quad b \quad \text{for the beta distribution of the system reliability are functions of the subsystem reliabilities.} \]

To demonstrate the same reliability at the same confidence level for a zero-failure test, the number of test samples should be the same. Therefore, in Eqn. (14) and (15), we can set \( n_s = n \). Then we have:

\[ E(\hat{R}_i) = \frac{n}{n+1} R^* \]  \hspace{1cm} (27)

\[ \text{Var}(\hat{R}_i) = \frac{n}{n+1} - \left(\frac{n}{n+1}\right)^2 \]  \hspace{1cm} (28)

where \( n \) is the required samples for demonstrating the allocated reliability of each subsystem and \( k \) is the number of subsystems/components in a serial system. Substituting Eqn. (27) and (28) in Eqn. (23) and (24), we get:

\[ a = \left( \frac{n}{n+1} \right)^k \left( \frac{\left(\frac{n}{n+1}\right)^k - \left(\frac{n}{n+1}\right)^2}{\left(\frac{n}{n+1}\right)^k - \left(\frac{n}{n+1}\right)^2} \right) \]  \hspace{1cm} (29)

\[ b = \left( 1 - \frac{n}{n+1} \right)^k \left( \frac{\left(\frac{n}{n+1}\right)^k - \left(\frac{n}{n+1}\right)^2}{\left(\frac{n}{n+1}\right)^k - \left(\frac{n}{n+1}\right)^2} \right) \]  \hspace{1cm} (30)

It can be seen that the optimization is much simpler since it only needs to solve for one variable, \( n \). We need to find \( n \) in order to make \( \text{Inv Beta}(1-\text{CL}, a, b) \) equal to \( R_i^* \) as close as possible.

Assuming that the required system reliability \( R_i^* \) is 0.95 at a confidence level of 90%, the following table gives the optimization results of the allocated reliability for each subsystem for systems with different numbers of subsystems.

<table>
<thead>
<tr>
<th>Number of Subsystems</th>
<th>Allocated R at CL=90%</th>
<th>Allocated R w/o CL</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.9701</td>
<td>0.9747</td>
</tr>
<tr>
<td>3</td>
<td>0.9781</td>
<td>0.9830</td>
</tr>
<tr>
<td>4</td>
<td>0.9825</td>
<td>0.9873</td>
</tr>
<tr>
<td>5</td>
<td>0.9853</td>
<td>0.9898</td>
</tr>
<tr>
<td>6</td>
<td>0.9873</td>
<td>0.9915</td>
</tr>
<tr>
<td>7</td>
<td>0.9888</td>
<td>0.9927</td>
</tr>
<tr>
<td>8</td>
<td>0.9900</td>
<td>0.9936</td>
</tr>
</tbody>
</table>

Table 2- Allocated Reliability with Confidence Level for Serial Systems with Different Numbers of Subsystems

The last column in Table 2 is the result from the traditional equal reliability allocation without considering matching the confidence level requirement. For this example, the traditional equal allocation method gives more conservative results than the one from the method in this paper. For example, for a system with 8 subsystems, the table shows that traditional allocation requires each subsystem's reliability to be 0.9936. The result using the proposed method is a reliability of 0.9900 at a confidence level of 90%.

3 \textit{ILLUSTRATIVE EXAMPLES}

The following system will be used for illustration.

![Figure 1- System with Serial and Parallel Subsystems](image)

The required system reliability is 95% at a confidence level of 90%. Determine the required reliability for each subsystem at a confidence level of 90%. Assume a zero-failure test is required for demonstrating the reliability for each subsystem.

First, we need to decompose the system into a simple serial system. It is

![Figure 2- System Decomposition](image)

The mean and the variance of the reliability of block A will be from Eqn. (18) and Eqn. (19). They are:
\begin{equation}
E(\hat{R}_i) = 1 - \prod_{i=1}^n E(\hat{R}_i) = 1 - \prod_{i=1}^n \frac{1}{(n_i + 1)(n_i + 2)} - 1 + E(\hat{R}_i)
\end{equation}
\begin{equation}
Var(\hat{R}_i) = \left[ 1 - E(\hat{R}_i) \right] \left[ \frac{4}{(n_i + 2)^2(n_i + 2)} - 1 + E(\hat{R}_i) \right]
\end{equation}

The system reliability is:

\begin{equation}
E(\hat{R}_s) = E(\hat{R}_1) E(\hat{R}_2) E(\hat{R}_3) E(\hat{R}_4) = \frac{n_1 + n_2 + n_3 + n_4}{(n_1 + 1)(n_2 + 1)(n_3 + 1)(n_4 + 1)}
\end{equation}

From Eqn. (15), the variance of the estimated system reliability is:

\begin{equation}
Var(\hat{R}_s) = \prod_{i=1,2,3,4} \left[ E^2(\hat{R}_i) + Var(\hat{R}_i) \right] - \prod_{i=1,2,3,4} E^2(\hat{R}_i)
\end{equation}

where $E(\hat{R}_i)$ and $Var(\hat{R}_i)$ are given in Eqns. (31) and (32), $E(\hat{R}_i) = n_i / (n_i + 1)$, and $Var(\hat{R}_i) = n_i / (n_i + 1)^2 (n_i + 2)$ for $i = 3, 4$ based on Eqn. (12) and (13).

From $E(\hat{R}_i)$ and $Var(\hat{R}_i)$, the distribution parameters $a$ and $b$ for $\hat{R}_i$ are calculated from Eqn. (23) and (24). From Eqn. (25), we know:

\begin{equation}
\int R_d \text{Beta}(R_s; a, b) dR_s = 1 - CL = 0.1
\end{equation}

$a$ and $b$ are functions of $n_i$. We need to solve for $n_i$ using Eqn. (35) to make $R_{ul}$ as close to the required reliability $R_i$ as possible. Once $n_i$ is found, for a zero-failure test, $\hat{R}_i$ at a given confidence level $CL$, can be calculated from:

\begin{equation}
\hat{R}_i = \exp\left( \ln(1 - CL) / n_i \right)
\end{equation}

In allocating reliability to each subsystem, several practical factors should be considered. 1) Enough samples, cost, and test time should be allowed for each subsystem to demonstrate the allocated reliability requirement. 2) High risk subsystems should have higher reliability. In the following, we will use two examples to illustrate how to allocate reliability with these constraints.

3.1 Cost Based Allocation

For the example given in Figure 1, the system reliability is required to be 95% at a confidence level of 90%. Assume the cost of testing each subsystem is different, and limited samples are available to test for each subsystem. This information is given below.

<table>
<thead>
<tr>
<th>Component</th>
<th>Cost/Per Sample</th>
<th>Available Samples</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$100</td>
<td>40</td>
</tr>
<tr>
<td>2</td>
<td>$150</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>$250</td>
<td>No Limit</td>
</tr>
<tr>
<td>4</td>
<td>$900</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 3 - Cost and Available Samples for Each Subsystem

The total cost for the demonstration tests at the subsystem level should be controlled. The reliability allocation becomes an optimization issue as:

\text{Minimize: } Cost(n_1, n_2, n_3, n_4) = 100n_1 + 150n_2 + 250n_3 + 900n_4

\text{subject to: } n_i \leq 40, n_2 \leq 20, n_i \leq 100

\begin{equation}
R_{ul} = \text{Inv. Beta}(0.1, a, b) \geq R_i, 1 - CL = R^*_i
\end{equation}

\begin{equation}
a = f_a(R_1, R_2, ..., R_k), b = f_b(R_1, R_2, ..., R_k)
\end{equation}

If we set $CL = 0.9$ and use a zero-failure test for each subsystem, then the optimal solutions are:

<table>
<thead>
<tr>
<th>Component</th>
<th>Test Cost</th>
<th>Test Samples</th>
<th>CL</th>
<th>Allocated Reliability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$3,600</td>
<td>36</td>
<td>0.9</td>
<td>0.9380</td>
</tr>
<tr>
<td>2</td>
<td>$3,000</td>
<td>20</td>
<td>0.9</td>
<td>0.8913</td>
</tr>
<tr>
<td>3</td>
<td>$26,750</td>
<td>107</td>
<td>0.9</td>
<td>0.9787</td>
</tr>
<tr>
<td>4</td>
<td>$56,700</td>
<td>63</td>
<td>0.9</td>
<td>0.9641</td>
</tr>
<tr>
<td>Total Cost</td>
<td>$90,050</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4 - Allocated Reliability Based on Cost

The allocated subsystem reliability at a confidence level of 90% will meet the system reliability requirement. At the same time, the above allocation minimizes the total test cost.

3.2 Risk Based Allocation

In certain regulated industry (medical device, aerospace, food, drug, etc.), safety is more important than cost. Risk critical components should have higher reliability than components that are less important. One way to quantify risk is:

\begin{equation}
Risk_i = (1 - R_i) \times Cost_i
\end{equation}

where $Cost_i$ is the cost of a failure of subsystem $i$, representing the severity of failure consequence. For the system in Figure 1, assume the costs for each failure event are:

<table>
<thead>
<tr>
<th>Component</th>
<th>Cost of Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$100</td>
</tr>
<tr>
<td>2</td>
<td>$200</td>
</tr>
<tr>
<td>3</td>
<td>$3,000</td>
</tr>
<tr>
<td>4</td>
<td>$5,000</td>
</tr>
</tbody>
</table>

Table 5- Cost of Each Failure Event

\{1, 2\} means both component 1 and 2 failed during a mission. The reliability allocation becomes an optimization issue as:

\text{Minimize: } Risk = 100(1 - R_1) + 200(1 - R_2) + 3000(1 - R_3) + 5000(1 - R_4) + 2000(R_1 + R_2 - R_3 - R_4)

\text{subject to: } n_i \leq 100 \text{ (i=1,2,3,4)}

\begin{equation}
R_{ul} = \text{Inv. Beta}(0.1, a, b) = 0.95, 1 - CL = R^*_i
\end{equation}

\begin{equation}
a = f_a(R_1, R_2, ..., R_k), b = f_b(R_1, R_2, ..., R_k)
\end{equation}

Constraints of sample size ($n_i \leq 100$) are added to the optimization problem and the calculated system reliability is set to the target of 0.95. Without these constraints, the allocated reliability for each component will be 1, which gives the lowest risk of 0. The optimal solution is:
There are many different ways to define risk. For medical devices, a semi-quantitative method is given in BS EN ISO14971:2012. Different industries can easily apply the proposed reliability allocation method by defining their own objective functions.

### 3.3 Further Discussion

If one uses the allocated component reliability at a confidence level of 90% in Table 4 or 6 to calculate the system reliability, the result can be lower than the required reliability. For example using Table 6, we get:

\[
R_s = (R_1 + R_2 - R_1 R_2) R_3 R_4 = 0.9409
\]

which is lower than the required reliability of 0.95 at 90%. Is this right? To understand this, we need to look more deeply into the function of random variables. The system reliability is calculated from independent component reliabilities which are random variables. Define \(X_1\) and \(X_2\) as independent standard normal random variables and:

\[
X = X_1 + X_2
\]

We know the 90th percentile of \(X_1\) and \(X_2\) is 1.2816. \(X\) is also a normal distribution with mean of 0 and standard deviation of \(\sqrt{2}\), so its 90th percentile is 1.8124. Clearly, 1.8124 \(\neq\) 1.2816 + 1.2816 \(\approx\) 2.5631. Therefore, the sum of the percentiles of random variables is not the same as the percentile of the sum of random variables. For a system with a serial reliability configuration, we know:

\[
R_s = \prod_{i=1}^{k} R_i \Rightarrow \ln(R_s) = \sum_{i=1}^{k} \ln(R_i)
\]

The logarithm of the system reliability is the sum of the logarithm of the component reliabilities. Therefore, when we apply the traditional equal reliability allocation, the allocated reliability \(R_i\) does not have the same confidence level as the required system reliability. This is proved by the result in Table 2.

### 4 CONCLUSIONS

In this paper, we proposed a method for allocating a system reliability requirement with a given confidence level. The proposed method formulates the allocation problem as an optimization issue. It is a general method and can be applied to systems with serial, parallel, and complex reliability configurations. All the constraints such as the cost, time, and other factors can be easily handled by the proposed method. Two examples are included in this paper to show how the method can be applied.

### REFERENCES


### BIOGRAPHIES

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