Quantifying the Effect of a Potential Corrective Action on Product Life

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SUMMARY & CONCLUSIONS

When a product fails more often than desired, it is necessary to weigh the potential benefits of a corrective action with the costs associated with performing the corrective action in order to determine an appropriate course of action. One thing that should be considered is the effect of a proposed corrective action on product life; however, the business decision about whether to proceed with a corrective action often must be made before a new prototype can be produced and tested. Therefore, it is important to understand the approaches available to account for the effect of a potential corrective action on product life.

The approach most commonly used by life data analysts is to multiply the scale parameter of the life distribution by a predefined factor while leaving the shape parameter unchanged. The underlying assumption of such an approach is that the same failure rate behavior (i.e., failure mode) will occur after the corrective action, but the failures will happen at longer lives than previously observed.

An alternative approach is to consider each failure in the data set as a fractional failure. In this case, the failure is given a non-integer coefficient that represents the amount of failure left after corrective action, while the remainder of the failure is considered to be a suspension. This approach is analogous to using an effectiveness factor in reliability growth analysis. The effectiveness factor represents the failure intensity removed from the system due to a corrective action on a particular failure mode.

This paper investigates these two approaches for accounting for the effect of a potential corrective action on parameter estimates. Using both complete and right censored data described by a Weibull life distribution and analyzed using maximum likelihood parameter estimation, a relationship between the two methods is presented.

1 INTRODUCTION

Time to market is becoming increasingly important to companies in a variety of industries, especially as consumers' demand for the most up to date technology grows. One consequence is that managers are forced to make decisions before sufficient data becomes available. In particular, consider the case where a component which is necessary for a new system feature is found to have insufficient reliability for the system to meet its reliability target. Because of the pressure to put a product on the market, the manager must decide quickly whether it makes more sense to try to improve the component through a redesign or other corrective action, or to abandon incorporation of the new feature into the product. Since there is not enough time to develop and test a new component before the decision must be made, the reliability improvement expected as a result of the corrective action must be estimated without test data to analyze.

One way to estimate the improvement that could be made by performing a corrective action is to use engineering judgement or other tool (such as computer aided engineering) to estimate how much longer, on average, a component will survive after the corrective action. This estimate can be used as a scale factor. By combining the life distribution obtained by data analysis of the current component and the scale factor, a life distribution governing the component after the corrective action has been taken can be defined [1].

A second way to estimate the improvement due to a potential corrective action is to again use engineering judgement or other tool to estimate the effectiveness of the corrective action. Starting with the life data set for the current component, the corrective action effectiveness estimate is used to manipulate the data set. Then the manipulated data set is analyzed to determine the life distribution of the component after corrective action [2].

The purpose of this paper is to determine if there is a mathematical relationship between the value of the scale factor and the value of the corrective action effectiveness. Background and a simple example are presented to illustrate the basic concepts of how to apply each method to estimate improvement after a corrective action. Then, a relationship between the scale factor and the corrective action effectiveness is proposed and verified for a Weibull distribution [3] using a wide variety of data sets generated using the Monte Carlo method.

2 BACKGROUND

The two methods of estimating an improvement in product life due to a corrective action are presented in this section. Each method is based on maximum likelihood estimation, which was developed by Sir R. A. Fisher [4]. The derivations of the likelihood functions for each method are presented in this section are provided in the Appendix.

2.1 Scale Factor Method

The first method is to multiply the scale parameter for the data set by a scale factor, c, while leaving the shape parameter unchanged. This method assumes that the product will have a longer life but that it will have the same failure rate behavior, i.e., failure mode, after the corrective action is implemented. The value of the scale factor can be any number between one and infinity. A scale factor of one corresponds to no improvement in the product, while a scale factor of infinity corresponds to complete elimination of the failures from the product. From a practical perspective, it is unlikely that a proposed corrective action will extend the life of the product by more than a factor of three, so this paper will consider scale factors greater than one and less than or equal to three.

Consider the data set shown in Table 1. Assuming a Weibull failure distribution, the likelihood function for this data set is

$$L = \prod_{i=1}^{N_F} \frac{\beta}{\eta}\left(\frac{t_i}{\eta}\right)^{\beta-1} exp\left[-\left(\frac{t_i}{\eta}\right)^{\beta}\right] \prod_{i=1}^{N_S} exp\left[-\left(\frac{t_j}{\eta}\right)^{\beta}\right] \quad (1)$$

Using maximum likelihood estimation, the shape parameter, β, for this data set is 2.986 and the scale parameter, η, is 374.3 hours. Suppose that it was assumed that the life of the product would increase by 50% after corrective action. This corresponds to a scale factor, c, of 1.5. The likelihood function for this case is

$$L = \prod_{i=1}^{N_F} \frac{\beta}{\eta}\left(\frac{ct_i}{\eta}\right)^{\beta-1} exp\left[-\left(\frac{ct_i}{\eta}\right)^{\beta}\right] \prod_{i=1}^{N_S} exp\left[-\left(\frac{ct_j}{\eta}\right)^{\beta}\right] \quad (2)$$

Therefore, the shape parameter after redesign would be 2.986 and the scale parameter would be 561.4 hours. Note that this result also could be obtained by multiplying all failure and suspension times by the scale factor and calculating parameters based on equation (1) using the scaled data set.

Table 1 – Simple Data Set with Failures and Suspensions

State	Time (h)
Failure	200
Failure	320
Suspension	400

2.2 Fractional Failure Method

The second method is to consider each failure time in the data set to be a fractional failure. The expected effectiveness of the corrective action, $(1-a)\%$, is estimated as a value between 0% and 100%. In this case, a 0% effectiveness corresponds to no improvement in life while a 100% effectiveness corresponds to complete elimination of the failures from the product. Then, each failure in the data set is split into two data points, one failure point with a weight of $a\%$ and one suspension with a weight of $(1-a)\%$. This approach mimics the use of the effectiveness factor to represent the amount of failure intensity removed from the system by a corrective action in repairable systems analysis, where an analysis of historical data suggests that an effectiveness factor of 0.7 is typical [5].

Again consider the data set shown in Table 1. Suppose that it was assumed that the corrective action effectiveness was 70%. This corresponds to a value of a of 30%. Therefore, the data in Table 1 would be adjusted to become the data in Table 2. Assuming a Weibull failure distribution, the likelihood function is

$$L = \prod_{i=1}^{N_F} \left[\frac{\beta}{\eta}\left(\frac{t_i}{\eta}\right)^{\beta-1}\right]^{a} exp\left[-\left(\frac{t_i}{\eta}\right)^{\beta}\right] \prod_{i=1}^{N_S} exp\left[-\left(\frac{t_j}{\eta}\right)^{\beta}\right] \quad (3)$$

Using maximum likelihood estimation, the shape parameter, β, for this data set is 2.986 and the scale parameter, η, is 560.1 hours. The results are summarized in Table 3 below. Notice that both methods of accounting for the impact of the proposed corrective action used different inputs, yet produced nearly identical results.

Table 2 – Simple Data Set After Incorporation of Fractional Failures

Number in State	State	Time (h)
0.3	Failure	200
0.7	Suspension	200
0.3	Failure	320
0.7	Suspension	320
1	Suspension	400

Table 3 – Comparison of Results Using Raw Data, a Scale Factor of 1.5, and a Corrective Action Effectiveness of 70%

Weibull Distribution Parameter	Raw Data	Scale Factor	Corrective Action Effectiveness
β	2.896	2.896	2.896
η (h)	374.3	561.4	560.1

For an examination of the effect of different estimates of scale factor and corrective action effectiveness on predicted product life, see Honecker, et. al. [6].

3 RELATIONSHIP BETWEEN THE SCALE FACTOR AND THE CORRECTIVE ACTION EFFECTIVENESS

In order to determine a relationship between the two methods of accounting for the effect of a proposed corrective action on product life, the equations that result from taking the partial derivative of the log-likelihood function with respect to the scale parameter are compared. For the case of applying a scale factor, the equation is

$$-\frac{N_F\beta}{\eta} + \left(c^{\beta}\right)\left(\frac{\beta}{\eta}\right)\left[\sum_{i=1}^{N_F}\left(\frac{t_i}{\eta}\right)^{\beta} + \sum_{j=1}^{N_S}\left(\frac{t_j}{\eta}\right)^{\beta}\right] = 0 \quad (4)$$

For the case of applying a corrective action effectiveness value, it is

$$-\frac{N_F\beta}{\eta} + \left(\frac{1}{a}\right)\left(\frac{\beta}{\eta}\right)\left[\sum_{i=1}^{N_F}\left(\frac{t_i}{\eta}\right)^{\beta} + \sum_{j=1}^{N_S}\left(\frac{t_j}{\eta}\right)^{\beta}\right] = 0 \quad (5)$$

The only difference between the two equations is the first factor in the second term. If these two factors are set equal to each other, then one can write the complement of the corrective action effectiveness, a, in terms of the scale factor, c, as

$$a = c^{-\beta} \quad (6)$$

Since there is no closed form solution for obtaining the shape and scale parameters using maximum likelihood estimation, examples will be used to verify the relationship between the methods for incorporating the effect of a potential corrective action.

Monte Carlo simulation was used to generate data sets of various sizes and combinations of failures and suspensions in order to investigate whether the approximation of equation (6) is valid for values of scale factor of practical importance (i.e., greater than one and up to three). There were four categories of data sets considered:

- 10 exact times to failure,
- 100 exact times to failure,
- 50 failures and 50 suspensions, with all suspensions at the end of the data set, and
- Approximately 50 failures and 50 suspensions, with the suspensions randomly scattered among the failure times.

For each category, the following three data sets were generated:

- Weibull shape parameter less than one,
- Weibull shape parameter equal to one, and
- Weibull shape parameter greater than one.

For each data set, the following three scale factors were used:

- 1.5
- 2.0
- 3.0

There was a total of 36 combinations of failures, Weibull shape parameters, and scale factors considered. For each combination, the following procedure was used:

1. The shape and scale parameters for the data set were calculated assuming a Weibull distribution.
2. The scale parameter was multiplied by the scale factor to obtain the value of the scale parameter after the proposed corrective action. This value is reported as η_c.
3. The complement of the corrective action effectiveness, a, was calculated using the scale factor and the shape parameter.
4. The data was adjusted using the calculated value of a as described in the Fractional Failure Method section. The new shape and scale parameters were calculated. In all 36 cases studied, the shape parameter matched the shape parameter calculated using the unadjusted data. The new scale parameter was reported as $\eta_{\alpha,}$.

Table 4, Table 5, Table 6, and Table 7 contain the results of these calculations. All results are in hours.

Table 4 – Results for Data Sets with 10 Exact Times to Failure

	$\beta = 0.389$ $\eta = 1043$			$\beta = 1.003$ $\eta = 1059$			$\beta = 2.456$ $\eta = 874$		
c	η_c	a	η_α	η_c	a	η_α	η_c	a	η_α
1.5	1565	0.854	1565	1589	0.666	1589	1311	0.369	1311
2.0	2087	0.764	2087	2118	0.499	2118	1748	0.182	1748
3.0	3131	0.652	3131	3178	0.332	3178	2622	0.0673	2622

Table 5 – Results for Data Sets with 100 Exact Times to Failure

	$\beta = 0.404$ $\eta = 754$			$\beta = 1.009$ $\eta = 1019$			$\beta = 2.600$ $\eta = 1036$		
c	η_c	a	η_α	η_c	a	η_α	η_c	a	η_α
1.5	1131	0.849	1131	1529	0.664	1529	1554	0.348	1554
2.0	1508	0.756	1508	2038	0.497	2038	2072	0.165	2072
3.0	2262	0.642	2262	3057	0.330	3057	3108	0.0575	3108

Table 6 – Results for Data Sets with 50 Exact Times to Failure and 50 Suspensions at the End of the Data Set

	$\beta = 0.381$ $\eta = 996$			$\beta = 1.003$ $\eta = 987$			$\beta = 2.486$ $\eta = 1050$		
c	η_c	a	η_α	η_c	a	η_α	η_c	a	η_α
1.5	1495	0.857	1495	1480	0.666	1480	1575	0.365	1575
2.0	1993	0.768	1993	1974	0.499	1974	2100	0.178	2100
3.0	2989	0.658	2989	2961	0.332	2961	3150	0.0650	3150

Table 7 – Results for Data Sets with 100 Data Points with Approximately 50% Suspensions Scattered Randomly Throughout Data Set

	$\beta = 0.371$ $\eta = 1058$			$\beta = 1.006$ $\eta = 1152$			$\beta = 2.469$ $\eta = 1108$		
c	η_c	a	η_α	η_c	a	η_α	η_c	a	η_α
1.5	1587	0.862	1587	1728	0.665	1728	1661	0.367	1661

2.0	2116	0.776	2116	2304	0.498	2304	2215	0.181	2215
3.0	3174	0.669	3174	3456	0.331	3456	3323	0.0664	3323

For every combination of failures, Weibull shape parameters, and scale factors considered, the values of the scale parameter obtained by the scale factor method and the fractional failure method were identical. Thus, the relationship proposed in equation (6) is valid for the range of scale factors of considered.

4 CONCLUSIONS

Two methods for incorporating the effect of a proposed corrective action, the scale factor method and the fractional failure method, were examined. First, life distribution parameters were calculated for each method using a simple example. Then, a simple mathematical relationship between the scale factor and the corrective action effectiveness was proposed. The relationship was validated using 36 data sets covering a variety of failure rate behaviors, sizes, and censoring schemes.

5 APPENDIX

This appendix contains the likelihood functions and their partial derivatives for the scale factor and fractional failure methods.

5.1 Likelihood Equations for the Scale Factor Method

The likelihood function for a data set containing both exact times to failure and suspensions is given by

$$L = \prod_{i=1}^{N_F} f(ct_i) \prod_{j=1}^{N_S} R(ct_j) \quad \text{(A1)}$$

For a data set that is described by a Weibull distribution, the likelihood function becomes

$$L = \prod_{i=1}^{N_F} \frac{\beta}{\eta}\left(\frac{ct_i}{\eta}\right)^{\beta-1} exp\left[-\left(\frac{ct_i}{\eta}\right)^{\beta}\right] \prod_{i=1}^{N_S} exp\left[-\left(\frac{ct_j}{\eta}\right)^{\beta}\right] \quad \text{(A2)}$$

Therefore, the log-likelihood function is

$$\Lambda = N_F ln\left(\frac{\beta}{\eta}\right) + \sum_{i=1}^{N_F}\left[(\beta-1) ln\left(\frac{ct_i}{\eta}\right) - \left(\frac{ct_i}{\eta}\right)^{\beta}\right] - \sum_{j=1}^{N_S}\left(\frac{ct_j}{\eta}\right)^{\beta} \quad \text{(A3)}$$

Taking the partial derivative with respect to the shape parameter yields

$$\frac{\partial\Lambda}{\partial\beta} = \frac{N_F}{\beta} + \sum_{i=1}^{N_F}\left[ln(c) + ln\left(\frac{t_i}{\eta}\right)\right]\left[1 - \left(c^{\beta}\right)\left(\frac{t_i}{\eta}\right)^{\beta}\right] - \sum_{j=1}^{N_S}\left[ln(c) + ln\left(\frac{t_j}{\eta}\right)\right]\left(c^{\beta}\right)\left(\frac{t_j}{\eta}\right)^{\beta} \quad \text{(A4)}$$

Taking the partial derivative with respect to the scale parameter yields

$$\frac{\partial\Lambda}{\partial\eta} = -\frac{N_F\beta}{\eta} + \left(c^{\beta}\right)\left(\frac{\beta}{\eta}\right)\left[\sum_{i=1}^{N_F}\left(\frac{t_i}{\eta}\right)^{\beta} + \sum_{j=1}^{N_S}\left(\frac{t_j}{\eta}\right)^{\beta}\right] \quad \text{(A5)}$$

The maximum value of the likelihood function is the combination of shape and scale parameters that satisfies the following system of equations

$$\begin{cases} \frac{N_F}{\beta} + \sum_{i=1}^{N_F}\left[ln(c) + ln\left(\frac{t_i}{\eta}\right)\right]\left[1 - \left(c^{\beta}\right)\left(\frac{t_i}{\eta}\right)^{\beta}\right] - \sum_{j=1}^{N_S}\left[ln(c) + ln\left(\frac{t_j}{\eta}\right)\right]\left(c^{\beta}\right)\left(\frac{t_j}{\eta}\right)^{\beta} = 0 \\ -\frac{N_F\beta}{\eta} + \left(c^{\beta}\right)\left(\frac{\beta}{\eta}\right)\left[\sum_{i=1}^{N_F}\left(\frac{t_i}{\eta}\right)^{\beta} + \sum_{j=1}^{N_S}\left(\frac{t_j}{\eta}\right)^{\beta}\right] = 0 \end{cases} \quad \text{(A6)}$$

When the data set only contains exact times to failure, the system reduces to

$$\begin{cases} \frac{N_F}{\beta} + \sum_{i=1}^{N_F}\left[ln(c) + ln\left(\frac{t_i}{\eta}\right)\right]\left[1 - \left(c^{\beta}\right)\left(\frac{t_i}{\eta}\right)^{\beta}\right] = 0 \\ -\frac{N_F\beta}{\eta} + \left(c^{\beta}\right)\left(\frac{\beta}{\eta}\right)\sum_{i=1}^{N_F}\left(\frac{t_i}{\eta}\right)^{\beta} = 0 \end{cases} \quad \text{(A7)}$$

5.2 Likelihood Equations for the Fractional Failure Method

The likelihood function for a data set containing exact times to failure and suspensions is given by

$$L = \prod_{i=1}^{N_F}[f(t_i)]^{a}[R(t_i)]^{1-a} \prod_{j=1}^{N_S} R(t_j) \quad \text{(A8)}$$

$$L = \prod_{i=1}^{N_F}[\lambda(t_i)]^{a}[R(t_i)] \prod_{j=1}^{N_S} R(t_j) \quad \text{(A9)}$$

For a data set that is described by a Weibull distribution, the likelihood function becomes

$$L = \prod_{i=1}^{N_F}\left[\frac{\beta}{\eta}\left(\frac{t_i}{\eta}\right)^{\beta-1}\right]^{a} exp\left[-\left(\frac{t_i}{\eta}\right)^{\beta}\right] \prod_{i=1}^{N_S} exp\left[-\left(\frac{t_j}{\eta}\right)^{\beta}\right] \quad \text{(A10)}$$

Therefore, the log-likelihood function is

$$\Lambda = a\left\{N_F ln\left(\frac{\beta}{\eta}\right) + \sum_{i=1}^{N_F}\left[(\beta-1) ln\left(\frac{t_i}{\eta}\right) - \left(\frac{1}{a}\right)\left(\frac{t_i}{\eta}\right)^{\beta}\right] - \sum_{j=1}^{N_S}\left(\frac{1}{a}\right)\left(\frac{t_j}{\eta}\right)^{\beta}\right\} \quad \text{(A11)}$$

Taking the partial derivative with respect to the shape parameter yields

$$\frac{\partial\Lambda}{\partial\beta} = \alpha\left\{\frac{N_F}{\beta} + \sum_{i=1}^{N_F}\left[\ln\frac{t_i}{\eta}\right]\left[1 - \frac{1}{\alpha}\left(\frac{t_i}{\eta}\right)^{\beta}\right] - \sum_{j=1}^{N_S}\left[\ln\frac{t_j}{\eta}\right]\frac{1}{\alpha}\left(\frac{t_j}{\eta}\right)^{\beta}\right\} \quad \text{(A12)}$$

Taking the partial derivative with respect to the scale parameter yields

$$\frac{\partial\Lambda}{\partial\eta} = \alpha\left\{-\frac{N_F\beta}{\eta} + \left(\frac{\beta}{\alpha\eta}\right)\sum_{i=1}^{N_F}\left(\frac{t_i}{\eta}\right)^{\beta} - \sum_{j=1}^{N_S}\left(\frac{t_j}{\eta}\right)^{\beta}\right\} \quad \text{(A13)}$$

The maximum value of the likelihood function is the combination of shape and scale parameters that satisfies the following system of equations

$$\begin{cases} \frac{N_F}{\beta} + \sum_{i=1}^{N_F} \left[ln\left(\frac{t_i}{\eta}\right) \right] \left[1 - \left(\frac{1}{a}\right)\left(\frac{t_i}{\eta}\right)^{\beta} \right] \\ \quad - \sum_{j=1}^{N_S} \left[ln\left(\frac{t_j}{\eta}\right) \right] \left(\frac{1}{a}\right)\left(\frac{t_j}{\eta}\right)^{\beta} = 0 \\ -\frac{N_F \beta}{\eta} + \left(\frac{1}{a}\right)\left(\frac{\beta}{\eta}\right) \left[\sum_{i=1}^{N_F} \left(\frac{t_i}{\eta}\right)^{\beta} + \sum_{j=1}^{N_S} \left(\frac{t_j}{\eta}\right)^{\beta} \right] = 0 \end{cases} \quad \text{(A14)}$$

When the data set only contains exact times to failure, the system reduces to

$$\begin{cases} \frac{N_F}{\beta} + \sum_{i=1}^{N_F} \left[ln\left(\frac{t_i}{\eta}\right) \right] \left[1 - \left(\frac{1}{a}\right)\left(\frac{t_i}{\eta}\right)^{\beta} \right] = 0 \\ -\frac{N_F \beta}{\eta} + \left(\frac{1}{a}\right)\left(\frac{\beta}{\eta}\right) \sum_{i=1}^{N_F} \left(\frac{t_i}{\eta}\right)^{\beta} = 0 \end{cases} \quad \text{(A15)}$$

REFERENCES

1. ReliaSoft Corporation, "The 1-Parameter Weibull," http://www.reliawiki.org/index.php/The_Weibull_Distribution#The_1-Parameter_Weibull, accessed July, 2016.
2. ReliaSoft Corporation, "Fractional Failure Analysis in Weibull++," http://www.weibull.com/hotwire/issue175/hottopics175.htm, accessed July, 2016.
3. W. Weibull, "A Statistical Distribution of Wide Applicability," *J. Appl. Mech.*, 18, 293-297.
4. H. J. Larson, *Introduction to Probability Theory and Statistical Inference, Third Edition*, New York, John Wiley & Sons, 1982.
5. L. H. Crow, "Planning a Reliability Growth Program Utilizing Historical Data," *Reliability and Maintainability Symposium (RAMS), 2011 Proceedings – Annual*, 1-6, 24-27 January 2011, doi: 10.1109/RAMS.2011.5754446
6. S. L Honecker, D. J. Groebel, and A. Mettas, "Practical Methods for Analyzing Reliability Test Data in the Presence of an Unexpected Failure Mode," SAE Technical Paper 2016-01-0274, 2016, doi:10.4271/2016-01-0274.

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